Azarina S.V. Backward mean derivatives for the diffusion type processes

Consider L_1-random variable $\xi(t)$, $t \in [0,T]$ given on a certain probability space $(\Omega, \mathcal{F}, \mathsf{P})$ with values in $\mathbb{R}^n$. Denote by E_t^{ξ} the conditional expectation w.r.t.σ-algebra generated by the mapping $\xi(t) : \Omega \to \mathbb{R}^n$. Denote by $D_*\xi(t)$, $D_*D_*\xi(t)$ the backward mean derivatives of the process $\xi(t)$ at the instant t of the first and second order respectively. Second order backward mean derivative $D_*D_*\xi(t)$ of the process $\xi(t)$ is the backward mean derivative for the regression $D_*\xi(t)$. This vector field we denote by $F_*(t,x)$.

Let $\xi(t)$ be a diffusion type process i.e. solution of the stochastic differential equation of diffusion type

$$\mathrm{d}\xi(t) = a(t, \xi(\cdot))\mathrm{d}t + A(t, \xi(\cdot))\mathrm{d}w(t).$$

where coefficients a and A are adapted w.r.t. $\mathcal{P}_t^{\xi}$. Denote by $\sigma = AA^*$ and p_t the distribution of $\xi(t)$.

Theorem 1. The backward mean derivatives of $\xi(t)$ satisfy formulaes

$$D_*\xi(t) = E_t^{\xi}(a(t, \xi(\cdot))) - E_t^{\xi}\left(\frac{1}{p_t}\nabla_j(\sigma(t, \xi(\cdot))p_t)\right) \tag{1}$$

$$D_*D_*\xi(t) = E_t^{\xi}\left(\frac{\partial F_*(t, \xi(t))}{\partial t} + (F_*\nabla F_*)(t, \xi(t)) - \frac{1}{2}\nabla^2 F_*(t, \xi(t))\right). \tag{2}$$