Boguslavskaya E.V.[1], **Shishkina E. L.**[2] ([1]Brunel University, London, United Kingdom; [2]Voronezh State University, Voronezh, Russian Federation)
On a generalization of Wiener chaos.

Function $\mathcal{H}_\alpha(x,y)=y^{\frac{\alpha}{2}}e^{\frac{x^2}{4y}}D_\alpha\left(\frac{x}{\sqrt{y}}\right)$, $x\in\mathbb{R}$, $y>0$, $\alpha\in\mathbb{C}$. is called **the power-normalised parabolic cylinder function**. Here, D_α denotes the parabolic cylinder function of the form
$$D_\alpha\left(z\right)=\sqrt{\pi}2^{\frac{\alpha}{2}}e^{-\frac{z^2}{4}}\left(\frac{1}{\Gamma\left(\frac{1-\alpha}{2}\right)}\,{}_1F_1\left(-\frac{\alpha}{2};\frac{1}{2};\frac{z^2}{2}\right)-\frac{\sqrt{2}z}{\Gamma\left(-\frac{\alpha}{2}\right)}\,{}_1F_1\left(\frac{1-\alpha}{2};\frac{3}{2};\frac{z^2}{2}\right)\right).$$

Theorem. Let $c\neq 0$ be some real constant. The equation $\Gamma\left(-\frac{\alpha}{2}\right)=c\Gamma\left(\frac{1-\alpha}{2}\right)$, $c\in\mathbb{R}\backslash 0$ has infinitely many real positive non-integer roots. Suppose $\alpha_k>0$ and $\alpha_m>0$ with $k,m\in\mathbb{N}$ are real but not integer roots of $\Gamma\left(-\frac{\alpha}{2}\right)=c\Gamma\left(\frac{1-\alpha}{2}\right)$. Then, the functions $\{\mathcal{H}_{\alpha_k}(x,y)\}_{k\in\mathbb{N}}$ form an orthogonal set with respect to x on the interval $(0,\infty)$ for a fixed y, with weight function $\frac{1}{\sqrt{2\pi y}}e^{-\frac{x^2}{2y}}$:
$$\begin{cases}\langle\mathcal{H}_{\alpha_k},\mathcal{H}_{\alpha_m}\rangle_w=\frac{1}{\sqrt{2\pi y}}\int\limits_0^\infty\mathcal{H}_{\alpha_k}(x,y)\mathcal{H}_{\alpha_m}(x,y)e^{-\frac{x^2}{2y}}dx=0, \qquad \alpha_k\neq\alpha_m,;\\ \frac{1}{\sqrt{2\pi y}}\int\limits_0^\infty\mathcal{H}^2_{\alpha_k}(x,y)e^{-\frac{x^2}{2y}}dx=y^{\alpha_k}\frac{\psi\left(\frac{1-\alpha_k}{2}\right)-\psi\left(-\frac{\alpha_k}{2}\right)}{2\Gamma(-\alpha_k)}.\end{cases}$$
This result used for determination of the fractional Wiener chaos.