M. Ilolov, **J.S. Rahmatov, S.M. Lashkarbekov**(National Academy of Sciences of Tajikistan) **Martingale solution of fractional stochastic heat equation**. We consider in $\mathbb{R}^2$ a fractional multiplicative stochastic heat equation of the form

$$D_t^\alpha u = a\triangle u + \sigma u\dot{W}, \tag{1}$$

with the initial condition

$$u(0,x) = \varphi(x), \tag{2}$$

where u-formal solution, D_t^α-fractional derivative of order $\alpha, 0 < \alpha < 1$, a, σ-some numbers, $\triangle$-Laplace operator, $\dot{W}$-temporal-spatial white noise on $[0,\infty) \times \mathbb{R}^2$[1], $\varphi(x)$ a random field from $L^2(\Omega,\ \mathcal{F},\ \mathbb{P},\ \mathbb{R}^2)$. Let

$$p_t x = \frac{1}{2\pi t}\exp\left(-\frac{|x|^2}{2t}\right), t > 0, x \in \mathbb{R}^2$$

be the Gaussian density with zero mean and covariance tI (I-identity matrix).

Let us give a statement regarding the martingale problem for a measurable process $Z^{v,\varphi}$ related to (1)-(2).

Theorem 1. Let $\varphi \in C^+(\mathbb{R}^2), \psi \in C_b^2(\mathbb{R}^2)$ and $v \in \mathbb{R}$. Let $Z^{v,\varphi}(\psi) = \int_{\mathbb{R}^2}\psi(x)Z^{u,\varphi}(dx)$-continuous stochastic process. Then

$$M_t^{v,\varphi,\alpha} = Z_t^{v,\varphi}(\psi) - \int\limits_{\mathbb{R}^2\times\mathbb{R}^2}\varphi(x)\psi(x)dxdy - \frac{1}{\Gamma(1-\alpha)}\int\limits_0^t (t-s)^{-\alpha}Z_s(a\triangle\varphi)ds$$

is $\left\{\mathcal{F}^{z^v}\right\}_{t\geq 0}$ — martingale, such that

$$M_0^{v,\varphi,\alpha}(\psi) = 0$$

$$< M^{v,\varphi,\alpha}(\psi) >_t = -\lim_{\varepsilon\to 0}\frac{4\pi}{\Gamma(1-\alpha)\cdot log\ \varepsilon}\int\limits_0^t (t-s)^{-\alpha}\int\limits_{\mathbb{R}^2}\left(Z_s^{v,\varphi}(p_\varepsilon(\cdot - z))\right)^2\psi^2(z)dzds \tag{3}$$

where

$$Z_t^{v,\varphi}(p_\varepsilon(\cdot - z)) = \int\limits_{\mathbb{R}^2\times\mathbb{R}^2}\varphi(y)p_\varepsilon(z-x)Z_t^v(dy,dz)$$

and (3) is locally uniformly convergent in probability.

References

1. R.Y. Molay Hachemi, B. Øksendal, *The fractional stochastic heat equation driven by time-space white noise*, Fractional Calculus and Applied Analysis, 2023, 26, 513–532.