Multitype non-symmetric branching random walks on $\mathbf{Z}^d$ with a block-triangular branching intensity matrix

Irina Lukashova, PDMI

We consider a continuous-time multitype non-symmetric branching random walk on $\mathbf{Z}^d$ with periodically located branching sources. The intensity matrices of the random walk for particles of type k are denoted by $A_k = \{a_k(v,u)\}_{v,u\in\mathbf{Z}^d}$. The branching intensities from type l to type r are given by the matrix $Q_r^l = \operatorname{diag}\{\beta_r^l(v)\}_{v\in\mathbf{Z}^d}$, where $\beta_l^r(v) = 0$ for $l \le r$ and all $v \in \mathbf{Z}^d$.

Let $m_k^l(t,v,u)$ denote the mean number of particles of type k at time t at site u, given that the process started with a single particle of type l at site v. For each j, denote by $\lambda_1^j(0)$ the upper bound of the spectrum of the operator $A_j + Q_j^j$. The following statement holds.

Theorem 1. *Assume that $\lambda_1^r(0) \ne \lambda_1^s(0)$ for all $r,s \in \{1,\dots,n\}$ with $r \ne s$. Then for any $u,v \in \mathbf{Z}^d$ and any $l,k \in \{1,\dots,n\}$ there exists a constant $C = C(u,v,d,l,k) > 0$ such that*

$$m_k^l(t,v,u) = C \cdot \frac{e^{\max\limits_{l\le j\le k} \lambda_1^j(0)\, t}}{t^{d/2}}\bigl(1+o(1)\bigr), \quad t \to \infty.$$