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On the Cauchy Problem for a Class of Stochastic Nonlinear Wave Equations

We study the Cauchy problem for the Klein-Gordon equation with noise on the right-hand side:

$$u_t(x,t) - \psi(x) + \int_0^t u_{xx}(x,s)ds = \int_0^t F(u(s,x)) * dV'(x+s), \tag{1}$$

$u(x,0) = \varphi(x)$, $u_t(x,0) = \psi(x)$, where $F \in C(R)$ is the given nonlinearity, $\varphi(x), \psi(x)$ are the initial data, and the integral on the right-hand side is the symmetric integral over the process $V(s)$.

Theorem. *Let the process $V(s)$ be constant on any interval with probability 1. Then the solution to the Cauchy problem (1) has the form*

$$u(x,t) = \Phi\big(x-t,\ V(x+t)\big),$$

where $\Phi(\xi, y)$ satisfies the equation $\Phi_{\xi y} = -\frac{1}{4}F(\Phi)$ and the initial conditions

$$\Phi(\xi, V(\xi)) = \varphi(\xi), \quad \Phi_w(\xi, V(\xi)) = \beta(\xi), \quad \Phi_\xi(\xi, V(\xi)) = -\alpha(\xi),$$

if the function $\Psi(x) = \int_{x_0}^x \psi(y)dy$ has a stochastic differential with a symmetric integral $d\Psi(\xi) = \alpha(\xi)d\xi + \beta(\xi) \star dV(\xi)$.

Exact solutions of problem (1) are given for various right-hand sides of the equation; random kinks are constructed for the sine-Gordon equation.

REFERENCES

1. *Nasyrov F. S.* Local Times, Symmetric Integrals, and Stochastic Analysis, Moscow: FIZMATLIT, 2011.