Ornstein-Uhlenbeck Dynamics under Colored Noise

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The paper considers the problem of detecting anomalies in the Ornstein-Uhlenbeck process driven by fractional Brownian motion. Unlike the classical formulation, where the stochastic component is specified by a Wiener process, we propose using fractional Brownian motion B_t^H, parameterized by the Hurst exponent H. This makes it possible to study not only individual colored noises, but also a more general setting in which the degree of correlation and long-range memory of the noise component is determined by a continuous parameter.

Theorem. Let X_t be an Ornstein–Uhlenbeck process with a fractional noise component,

$$dX_t = \lambda(\mu - X_t)dt + \sigma dB_t^H,$$

and let its discretization be given by

$$X_{i+1} = X_i + \lambda(\mu - X_i)\Delta t + \sigma \Delta B_i^H.$$

Suppose that the state change points are determined by the PELT algorithm, and the task of predicting their neighborhoods is solved by machine learning models using the previous increments of the process. Then, for $H \neq 1/2$, the correlation structure of the fractional noise changes the statistical observability of transition states: in the presence of memory in the noise, the neighborhoods of transition points become more distinguishable for machine learning models than in the classical case $H = 1/2$, which corresponds to white noise.

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