A hierarchical discounted Vovk–Azoury–Warmuth forecaster for online regression in RKHS

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We study adversarial online regression with square loss in a reproducing kernel Hilbert space (RKHS) $\mathcal{H}$. For a comparator sequence $f_{1:T} = (f_1, \dots, f_T) \in \mathcal{H}^T$ define the dynamic regret by

$$R_T(f_{1:T}) = \sum_{t=1}^{T} \frac{1}{2}(\widehat{y}_t - y_t)^2 - \sum_{t=1}^{T} \frac{1}{2}(f_t(x_t) - y_t)^2.$$

The proposed H-VAW-D algorithm combines discounted Vovk–Azoury–Warmuth (DVAW) forecasters [1] with random features [2]. For each feature dimension m, it aggregates DVAW experts over discount factors γ. At the top level, VAW aggregates the resulting predictions over the grid of dimensions $m \in \{1, 2, 4, \dots, 2^{\lceil \frac{1}{2} \log_2 T \rceil}\}$.

Theorem 1 *Let $\mathcal{H}$ be an RKHS with reproducing kernel k, and assume that*

$$k(x, y) = \int_{\Theta} \varphi(x;\theta)\varphi(y;\theta)\, P(d\theta), \qquad |\varphi(x;\theta)| \le a.$$

Assume also that $|y_t|, |\widetilde{y}_t| \le Y$ and $\|f_t\|_{\mathcal{H}} \le R$ for $t = 1, \dots, T$. Let A be the H-VAW-D algorithm. Then,

$$\mathsf{E} R_T^A(f_{1:T}) = O\left(T^{2/3} P_T(f_{1:T})^{1/3} + \sqrt{T} \ln T\right),$$

where $P_T(f_{1:T}) = \sum_{t=1}^{T-1} \|f_{t+1} - f_t\|_{\mathcal{H}}$ is the path length. The expectation is taken with respect to the auxiliary randomness used to construct the feature maps, and the constant depends only on a, R, Y and fixed grid parameters.

REFERENCES

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