Rykov V. V. (Moscow, Russia)**Positive recurrence of Marked Markov Process for "m-out-of-n" reliability system.**

For a reliability system "m-из-n" study the Marked Markov Process $Z = (J; \mathbf{V})$ is used. Here $j \in \mathcal{J} = \{0, 1, \ldots, m\}$ is the number of failed elements, and marks $\mathbf{V} = (\mathbf{X}, \mathbf{Y})$ are the residual life times of working $\mathbf{X}$, and repaired $\mathbf{Y}$ elements, arranged in ascending order. It is supposed that elements' life and repair times $A_k, B_k\,(k = 1, 2, \ldots)$ are i.i.d, r.v.s with absolutely continuous cumulative distribution functions $A(\cdot)$, $B(\cdot)$ and two finite moments, for which the following assumptions hold.

Ass-1. The domain of values of distributions $A(\cdot)$, $B(\cdot)$ are the R_+ and its probability density functions are continuous.

Ass-2. Distributions $A(\cdot)$, $B(\cdot)$ are aging ones.

Such a process is determined constructively on a probability space $(\Omega, \mathcal{F}, \mathbf{P})$ of sequences of r.v.s A_k, B_k, $k = 1, 2, \ldots$, and takes the values in subspace $\mathcal{J} \times E$ of a direct product Euclidean spaces $R^1 \times R^l$ with appropriate Borel σ-algebras. By usual method the process transformations generate the process distributions

$$\mu_k(j; \Gamma_j) = \mathbf{P}\{J_k = j; \mathbf{V}_k \in \Gamma_j\}, \;\; \pi_k(j) = \mu_k(j; E_j) \quad (j \in {}_j, \; \Gamma_j \in E_j).$$

To study the process Z stationary regime we propose conditions of its irreducibility and positive recurrence.

Theorem 1. Under condition **Ass-1** the process Z is irreducible one, i.e. for any state $(i; \mathbf{v}_i)$ any subset of states $(j; \Gamma_j)$ of positive probability is attainable with positive probability in a finite number k of steps

$$P_{ij}(\mathbf{v}_i, \Gamma_j) \equiv \mathbf{P}\{J_k = j; \mathbf{V}_k \in \Gamma_j \mid J_0 = i; \mathbf{V}_0 = \mathbf{v}_i\} > 0. \qquad (1)$$

To prove the process Z stationary regime existence we check the conditions of Prochorov's theorem (see, for example [1]) about tightness of its family probability distribution $\mu_k(\cdot)$.

Theorem 2. Under conditions **Ass-1, Ass-2** the family of the process Z distributions $\mu_k(\cdot)$ is tight, i.e. for any $\varepsilon > 0$ and all $j \in \mathcal{J}$ there are exist bounded sets $C_j \in \mathcal{E}_j$ such that

$$\mu_k(j; C_j) \geq \pi_k(j)(1-\varepsilon), \quad \text{and} \quad \mu_k(C) = \sum_{j \in \mathcal{J}} \mu_k(j; C_j) \geq 1-\varepsilon \quad \forall\, j \in \mathcal{J}, \; k = 1, 2, \ldots. \qquad (2)$$

References

[1] Shiryaev A.N. Probability. M: "Nauka", Phiz.-math. lit., 1980, 576p.