

Slepov N. A. (Moscow, Russia). **The convergence rate of maximum likelihood estimators.**

A number of articles [1-3] are devoted to the asymptotic properties of maximum likelihood estimators and approach on this issue is based on Stein's method. The obtained results permit to clarify and simplify some inequalities related to the proximity of maximum likelihood estimators to the estimated parameter. In particular, the inequality for distributions from the exponential family is refined.

Consider an independent and identically distributed (i.i.d.) sample of observations $\mathbf{X} = (X_1, \dots, X_n)$ with density $f(x|\theta_0)$, $\theta_0 \in \Theta \subset \mathbf{R}$. Let $\hat{\theta}_n(\mathbf{X})$ be the maximum likelihood estimator of parameter. (\mathbf{A}) denotes the set of regularity conditions guaranteeing the existence, uniqueness and asymptotic normality of such estimators. The mentioned conditions are traditional and are given, for example, in [1, p. 155]. The set of conditions (\mathbf{A}) entails $U_n \xrightarrow{d} Z \sim N(0, 1)$, where $U_n = \sqrt{ni(\theta_0)}(\hat{\theta}_n(\mathbf{X}) - \theta_0)$, $i(\theta_0)$ is the expected Fisher information quantity.

Let us recall the probability density function for one-parameter exponential families is given by $f(x|\theta) = \exp\{k(\theta)T(x) - A(\theta) + S(x)\}\mathbf{I}_{\{x \in B\}}$, where the set $B = \{x : f(x|\theta) > 0\}$ does not depend on θ . Note that under conditions (\mathbf{A}) , derivatives of functions k and A up to the third order exist.

Theorem 1 *Let \mathbf{X} be the i.i.d. sample with density from exponential family and the set of conditions (\mathbf{A}) is satisfied. The function $D(\theta) := A'(\theta)/k'(\theta)$ is invertible. $\mathbb{E}|T(X_1) - D(\theta_0)|^3$ is finite. Also, there exist a constant $\varepsilon > 0$ with $(\theta_0 - \varepsilon, \theta_0 + \varepsilon) \subset \Theta$ for which*

$$C(\varepsilon, \theta_0) := \sup_{\theta \in (\theta_0 - \varepsilon, \theta_0 + \varepsilon)} |D''(\theta)| < \infty.$$

Then, for any absolutely continuous and bounded function h we have

$$\begin{aligned} |\mathbb{E}h(U_n) - \mathbb{E}h(Z)| \leq & \frac{1}{\sqrt{n}} \left[\|h'\| \frac{|k'(\theta_0)|^{3/2} \mathbb{E}|T(X_1) - D(\theta_0)|^3}{|D'(\theta_0)|^{3/2}} \right. \\ & \left. + \frac{(1 + \gamma_n)}{|D'(\theta_0)k'(\theta_0)|} \left(\frac{2\|h\|}{\varepsilon^2 \sqrt{n}} + \|h'\| \frac{\sqrt{|k'(\theta_0)|}}{2\sqrt{|D'(\theta_0)|}} C(\varepsilon, \theta_0) \right) \mathbb{I}_{\{D(\theta) \neq \theta\}} \right] \end{aligned}$$

where $\gamma_n = \gamma_n(\theta_0) = o(1)$ as $n \rightarrow \infty$, $\|\cdot\|$ - norm of L^∞ .

REFERENCES

1. *Anastasiou A., Ley C.* Bounds for the asymptotic normality of the maximum likelihood estimator using the Delta method. ALEA, Lat. Am. J. Probab. Math. Stat., 2017, vol. 14, p. 153–171.
2. *Anastasiou A., Gaunt R. E.* Wasserstein distance error bounds for the multivariate normal approximation of the maximum likelihood estimator. Electron. J. Stat., 2021, vol. 15, No. 2, p. 5758–5810.
3. *Bulinski A. V., Slepov N. A.* Asymptotic behavior of maximum likelihood estimators. Theory Probab. Appl., 2026, vol. 71, No. 2, p. 3–24.