Suchkova D. A. (UUST, Ufa, Russia). **On modeling solutions of the Cauchy problem for stochastic Korteweg-de Vries equations.**

Let $V(t)$, $t \in [0,T]$, be a random process with continuous realizations with probability 1. We study the Cauchy problem for a non-autonomous stochastic Korteweg-de Vries equation with noise in the nonlinear term $d(u)_t + uu_x * dV(t) + u_{xxx}dt = 0$, $u(x,0) = u_0(x)$ (**1**), $(x,t) \in R \times [0,T]$, the noise is represented as a symmetric integral [1],[2]. Let $S(t) = e^{-t\partial_x^3}$ be the linear Airy semigroup [3], $T_n = \{0 = t_0^{(n)} < t_1^{(n)} < \ldots < t_{M_n}^{(n)} = T\}$, $n \in N$ be the sequence of partitions of the segment $[0,T]$, $|T_n| := \max_j \big(t_j^{(n)} - t_{j-1}^{(n)}\big) \to 0$, and $V^{(n)}(t)$ be the broken approximation of the function $V(t)$ over the nodes of the partition T_n. $u^{(n)}$ – the corresponding solution $u_t^{(n)} + u_{xxx}^{(n)} + \dot{V}^{(n)}(t)u^{(n)}u_x^{(n)} = 0, u^{(n)}(0) = u_0$.

Theorem 1. *Let* $s > \frac{3}{2}$, $u_0(x) \in H^s(R)$, $V^{(n)}(t) \to V(t)$ *uniformly on* $[0,T]$, *and the family of solutions* $u^{(n)}$ *form a fundamental sequence in* $C([0,T]; H^{s-1}(R))$. *Then the limit* $u = \lim\limits_{n\to\infty} u^{(n)}$ *is an exact pathwise mild solution of (1) and is represented as* $u(t) = S(t)u_0 - \int_0^t S(t-\tau)\, u(\tau)u(\tau)_x * dV(\tau)$.

We introduce the linear flow $L_\tau(v) := S(\tau)v = e^{-\tau\partial_x^3}v$ and the nonlinear flow $N_\tau(v) := w(\tau), w_\tau + a\,w\,w_x = 0, w(0) = v$ [4], then one step of the Strang's numerical scheme is: $\Sigma_\tau = L_{\tau/2} \circ N_\tau \circ L_{\tau/2}$. Let $u(t_n)$ be the exact solution of (1) at time $t_n = n\tau$, and u^n be the numerical solution after n steps of the scheme.

Theorem 2. *Let* $s > \frac{3}{2}$, $0 = t_0 < \ldots < t_N = T$, $\Delta t_n = t_{n+1} - t_n$, $\Delta V_n = V(t_{n+1}) - V(t_n)$, *then the Strang's numerical scheme for (1) is represented as:* $u^{n+1} = L_{\Delta t_n/2}\, N_{\Delta V_n} L_{\Delta t_n/2} u^n, u^0 = u_0$ *and has the second order of approximation.*

Remark. In this work numerical methods for solving problem (1) are investigated.

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