Random walk of a particle system on a cylinder with accumulation

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Let $\mathbb{R}_+ \times [0; 2\pi)$ contain particles with coordinates (x_i, y_i) on $\mathbb{R}_+ \times [0; 2\pi)$, which form a Poisson point process with density λ. Each particle with coordinates $(x_i(t), y_i(t))$ at time t moves independently of the others according to the law

$$x_i(t) = x_i(0) - v(y_i)t + w_i^{(1)}(t),$$

$$y_i(t) = y_i(0) + w_i^{(2)}(t) \ (mod\ 2\pi)$$

where $v(y)$ is the drift, $\{y_i(0)\}$ is a set of constants, and $\{w_i^{(k)}(t)\}, k = 1, 2$ is a set of independent standard Wiener processes with zero mean. Upon collision with the boundary, particles are absorbed and disappear, with the boundary shifting by a certain amount proportional to δ, the particle's "size."

Now we formulate the main result. Let $N = N(t)$ be the number of particles absorbed by the boundary by time t, then $\xi = \xi(t) = \delta N(t)$ is the position of the boundary (along the x-axis) by time t.

Theorem 1.

Let $2\pi\lambda < \delta^{-1}$. If $v(y)$ is such that $\int_0^{2\pi} v(y)dy > 0$, then the boundary motion is asymptotically linear, i.e., as $t \to \infty$ a.s.

$$\frac{\xi(t)}{t} \to V = \overline{v}\frac{2\pi\delta\lambda}{1 - 2\pi\delta\lambda},\ \overline{v} = \frac{1}{2\pi}\int_0^{2\pi} v(y)dy$$

Literature

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