Ryadovkin K. S. (St. Petersburg, Russia). **Asymptotics of the Mean Number of Particles of Periodic Branching Random Walk at a Distant Point**

We consider a homogeneous Markov process taking values in the space of all finite integer-valued measures on $\mathbb{Z}^d$. Transitions in this process are interpreted either as a movement of a particle along the graph, or as a branching of a particle into some number of offspring at the same vertex where it was located. The evolution of particles is assumed to be independent of each other.

Let $\Gamma \subset \mathbb{Z}^d$ be a lattice generated by linearly independent integer vectors $g_1, \dots, g_d$. We assume that a particle at vertex v in a small time τ can, with probability $a(v,u)\tau + o(\tau)$, move to $u \neq v$, with probability $b_k(v)\tau + o(\tau)$ split into $k > 1$ offspring, with probability $b_0(v)\tau + o(\tau)$ die, and with probability $1 + a(v,v)\tau + b_1(v)\tau + o(\tau)$ nothing happens.

Let the matrix of transition intensities $(a(v,u))$ satisfy the conditions:

(i) $a(v,u) \geq 0$ for $v \neq u$, $a(v,v) < 0$;
(ii) $\sum_u a(v,u) = 0$;
(iii) $a(v,u) = a(u,v) = a(v+g, u+g)$ for all $g \in \Gamma$;
(iv) the graph with edges $\mathcal{E} = \{(v,u) : a(v,u) > 0\}$ is connected;
(v) the degrees of vertices are finite.

And the quantities $b_k(v)$ satisfy the conditions:

(a) $b_k(v) \geq 0$ for $k \neq 1$, $b_1(v) \leq 0$;
(b) $\sum_{k=0}^{\infty} b_k(v) = 0$;
(c) $\beta(v) = \sum_{k=1}^{\infty} k b_k(v) < \infty$;
(d) $\beta(v+g) = \beta(v)$ for $g \in \Gamma$ (periodicity).

Let $\lambda_1(\theta)$ be the largest eigenvalue of the matrix with entries $(\sum_{g \in \Gamma} a(v+g,u) e^{-i\langle g,\theta\rangle} + \delta_{vu}\beta(v))$. Fix an orthonormal basis in $\mathbb{R}^d$ in which the Hessian of the function $\lambda_1(\theta)$ is diagonal at zero. It turns out that the function $\lambda_1(\theta)$ is real-analytic in some neighborhood of zero, and zero is its global non-degenerate maximum. Then

$$\lambda_1(\theta) = \lambda_1(0) - \frac{1}{2}\sum_{j=1}^{d} a_j \theta_j^2 + o(|\theta|^2), \quad a_j > 0.$$

Denote by $M(v,u,t)$ the mean number of particles at vertex u at time t under the condition that at the initial time there was a single particle at point v. Let $0 \leq \alpha < \frac{5}{6}$, $s = (s_1, \dots, s_d) \in \mathbb{R}^d$, and let the observation point $u(t) \in \mathbb{Z}^d$ satisfy

$$u(t) = st^{\alpha} + o(t^{\min\{1/2,\, 1-\alpha\}}), \quad t \to +\infty.$$

Define the vector $h = (s_1/a_1, \dots, s_d/a_d)$.

Theorem. *Let the conditions above be satisfied. Then for the mean number of particles as $t \to +\infty$ the following holds:*

$$M(v,u(t),t) = e^{\lambda_1(-iht^{\alpha-1})t - \langle h,s\rangle t^{2\alpha-1}} t^{-\frac{d}{2}} C_t\big(1+o(1)\big),$$

where C_t can take a finite set of values.