Branching processes in non-favorable environment

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Let $\mathcal{Z} = \{Z_n,\ n = 0, 1, 2, ...\}$ be a critical branching process evolving in a random environment generated by a sequence $\{F_n(s), \quad s \in [0,1], n = 1, 2, ...\}$ of i.i.d. probability generating functions. Denote $X_i = \log F_i'(1), i = 1, 2, ...$ and introduce a random walk

$$S_0 = 0, \quad S_n = X_1 + ... + X_n,\ n \geq 1.$$

We impose the following restrictions on the characteristics of the process.

Assumption B1. The random variables $X_n, n = 1, 2, ...$ are independent and identically distributed with

$$\mathbf{E}X_1 = 0, \quad \sigma^2 = \mathbf{D}X_1 \in (0, \infty).$$

Besides, the distribution of X_1 is non-lattice.

Assumption B2. There is an $\varepsilon > 0$ such that

$$\mathbf{E}\left(\log^+ \frac{F_1''(1)}{\left(F_1'(1)\right)^2}\right)^{2+\varepsilon} < \infty.$$

Theorem 1 *Let Assumptions B1-B2 be valid. If $\varphi(n), n = 1, 2, ...$ is a sequence of positive numbers such that $\varphi(n) \to \infty$ as $n \to \infty$ and $\varphi(n) = o(\sqrt{n})$, then there is a constant $\Theta \in (0, \infty)$ such that*

$$\mathbf{P}\left(Z_n > 0; S_n \leq \varphi(n)\right) \sim \frac{\Theta\varphi^2(n)}{n^{3/2}}, \quad n \to \infty.$$

Theorem 1 compliments Theorem 1.1 in [1] where it was shown that there is a constant $C \in (0, \infty)$ such that $\mathbf{P}\left(Z_n > 0\right) \sim C\sqrt{n}$ as $n \to \infty$.

References

[1] Geiger, J., Kersting, G. The survival probability of a critical branching process in random environment. Theory Probab. Appl., 2001, 45:3, 517–525.