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Stochastic longitudinal oscillations viscoelastic rope with moving boundaries, taking into account damping forces

V. L. Litvinov (Russia, Samara)
Samara State Technical University,
Moscow State University
e-mail: vladlitvinov@rambler.ru

At present, reliability issues in the design of machines and mechanisms require more and more complete consideration of the dynamic phenomena that take place in the designed objects. The widespread use in technology of mechanical objects with moving boundaries necessitates the development of methods for their calculation. The problem of oscillations of systems with moving boundaries is related to obtaining solutions to integro-differential and partial differential equations in time-variable domains [1-10]. Such tasks are currently not well understood. Their peculiarity is the difficulty in using the known methods of mathematical physics, suitable for problems with fixed boundaries. The complexity of the solutions obtained is explained by the fact that up to now there has not been a sufficiently general approach to the analysis of the features of the dynamics of such systems. In connection with the danger of resonance, the study of forced oscillations is of great importance here. Attempts to investigate this process have been made, but the results obtained are limited mainly by a qualitative description of dynamic phenomena [1–4]. In addition, it is recognized that deterministic modeling of systems cannot be adequate for some types of problems, so it is necessary to switch to probabilistic-statistical, where there are random variables, stochastic fluctuations. When solving here, mainly approximate methods are used [5–9], since obtaining exact solutions is possible only in the simplest cases [10].

If the damping of transverse vibrations is mainly due to the action of external damping forces, then in the case of longitudinal vibrations, the damping is mainly affected by elastic imperfections in the material of the vibrating object [5-10]. The study of viscoelasticity includes the analysis of the stochastic stability of stochastic viscoelastic systems, their reliability, etc. The paper considers stochastic linear longitudinal oscillations of a viscoelastic rope with moving boundaries, taking into account the influence of damping forces. The case of a difference kernel makes it possible to reduce the problem of analyzing a system of stochastic integro-differential equations to the study of a system of stochastic differential equations. To estimate the expansion coefficients, it is proposed to apply the statistical numerical Monte Carlo method [11].

The differential equation describing the longitudinal vibrations of the rope (viscoelasticity is taken into account based on the Voigt hypothesis) has the form [7, 10]

$$U_{tt}(x,t)+2\alpha U_t(x,t)-a^2\left[U_{xx}(x,t)-\int_0^t K(t-\nu)U_{xx}(x,\nu)d\nu+\mu U_{xxt}(x,t)\right]=f(x,t). \tag{1}$$

Border conditions

$$U(v_0 t,t)=0;\ U(v_0 t+l_0,t)=0. \tag{2}$$

Initial conditions

$$U(x,0)=U_1(x);U_t(x,0)=0. \tag{3}$$

In problem (1) - (3) it is indicated: $U(x,t)$ – longitudinal displacement of the rope point with coordinate x at time t; $a^2 = E/\rho -$ velocity of wave propagation in the rope, $E-$ modulus of elasticity of the rope material, $\rho -$ linear mass density; $\alpha -$ resistance force of the medium acting per unit length of the rope, proportional to the speed of movement; $\mu -$ a small parameter that takes into account viscoelasticity; $v_0 t + l_0 -$ the law of motion of the rope boundary; $f(x,t) -$ a function that characterizes an external disturbance; $K(z) -$ relaxation core.

Let's introduce new variables that stop the bounds:

$$\xi = (x - v_0 t)/l_0; \quad \tau = at/l_0; \quad U(x,t) = V(\xi,\tau).$$

After transformations, we get:

$$V_{\tau\tau}(\xi,\tau) - 2vV_{\xi\tau}(\xi,\tau) - (1-v^2)V_{\xi\xi}(\xi,\tau) - 2k_0 V_{\xi}(\xi,\tau) + 2k_1 V_{\tau}(\xi,\tau) -$$

$$-d\int_{\xi}^{\xi+v\tau} K(-d(\xi-\eta))V_{\xi\xi}(\eta,\frac{1}{v}(\xi-\eta)+\tau)d\eta + \lambda\left(V_{\xi\xi\xi}(\xi,\tau) - \frac{1}{v}V_{\xi\xi\tau}(\xi,\tau)\right) = F(\xi,\tau); \tag{4}$$

$$V(0,\tau) = 0; \; V(1,\tau) = 0; \tag{5}$$

$$V(\xi,0) = V_1(\xi); \; V_{\tau}(\xi,0) = 0. \tag{6}$$

Here $v = \dfrac{v_0}{a}; \quad d = \dfrac{l_0}{v_0}; \quad k_0 = \alpha v l_0; \quad k_1 = \alpha v d; \quad \lambda = \dfrac{\mu}{d}; \quad F(\xi,\tau) = v^2 d^2 f(x,t).$

The function $F(\xi,\tau)$ can be represented as

$$F(\xi,\tau) = \sum_{n=1}^{\infty} F_n(\tau)\sin(\omega_n \xi), \; \omega_n = \pi n. \tag{7}$$

Theorem 1. The solution to problem (4)–(6) can be given as a string

$$V(\xi,\tau) = \sum_{n=1}^{\infty} V_n(\tau)\sin(\omega_n \xi). \tag{8}$$

Substituting (7), (8) into (4), after transformations, we obtain the system of equations

$$V_{n_{\tau\tau}}(\tau) + \left(2k_1 + \frac{\lambda}{v}\omega_n^2\right)V_{n_\tau}(\tau) + \omega_n^2(1-v^2)V_n(\tau) + \omega_n^2 d\int_{\xi}^{1} K(-d(\xi-\eta))V_n\left(\frac{1}{v}(\xi-\eta)+\tau\right)d\eta = F_n(\tau) \tag{9}$$

with initial conditions

$$V_n(0) = 2\int_0^1 V_1(\xi)\sin(\omega_n\xi)d\xi; \; V_{n_\tau}(0) = 0. \tag{10}$$

We accept the initial conditions and the external load as random, representing the sum of sinusoids with random amplitudes, denoting them $\tilde{V}(\xi)$ and $\tilde{F}(\xi,\tau)$ respectively. In this case, the oscillations will be random, and equations (9) form a system of random integro-differential equations

$$\tilde{V}_{n_{\tau\tau}}(\tau) + \left(2k_1 + \frac{\lambda}{v}\omega_n^2\right)\tilde{V}_{n_\tau}(\tau) + \omega_n^2(1-v^2)\tilde{V}_n(\tau) + \omega_n^2 d\int_{\xi}^{1} K(-d(\xi-\eta))\tilde{V}_n\left(\frac{1}{v}(\xi-\eta)+\tau\right)d\eta = \tilde{F}_n(\tau); \tag{11}$$

$$\tilde{V}_n(0) = 2\int_0^1 \tilde{V}_1(\xi)\sin(\omega_n\xi)d\xi; \; \tilde{V}_{n_\tau}(0) = 0. \tag{12}$$

Characteristics of random variables - mathematical expectation, variance and covariance, have the following form:

$$M\left(\tilde{V}(\xi,\tau)\right)=\sum_{n=1}^{\infty}M\left(\tilde{V}_n(\tau)\right)\sin\left(\omega_n\xi\right); \qquad (13)$$

$$D\left(\tilde{V}(\xi,\tau)\right)=\sum_{n,k=1}^{\infty}D_{n,k}\left(\tau\right)\sin\left(\omega_n\xi\right)\sin\left(\omega_k\xi\right); \qquad (14)$$

$$C\left(\tilde{V}(\xi,\tau,\zeta,\upsilon)\right)=\sum_{n,k=1}^{\infty}C_{n,k}\left(\tau,\upsilon\right)\sin\left(\omega_n\xi\right)\sin\left(\omega_k\zeta\right). \qquad (15)$$

To find the characteristics (13) - (15) of stochastic linear longitudinal oscillations of a viscoelastic rope, it is necessary to obtain statistical estimates for the solution of a system of random integro-differential equations (11). To do this, the relaxation kernel $K(z)$ can be taken in exponential form with a random component:

$$K(z,\bar{\beta})=K(z,\bar{b})\Big|_{\bar{b}=\bar{\beta}}=\sum_{j=1}^{N}c_j e^{-\beta_j z}, \qquad (16)$$

where $c_j \in R_+$, β_j – is a possible value of a positive random variable b_j.

Denote the dependence of $\tilde{V}(\xi,\tau)$ and $\tilde{V}_n(\tau)$ on the random vector $\bar{b}$ as $\tilde{V}(\xi,\tau,\bar{b})$ and $\tilde{V}_n(\tau,\bar{b})$, respectively. By changing the variable

$$u_{nj}(\tau,\bar{b})=\int_{\xi}^{1}e^{-\beta_j d\eta}\tilde{V}_n\left(\frac{1}{v}(\xi-\eta)+\tau\right),\bar{b})d\eta \qquad (17)$$

the system of random integro-differential equations (11) is transformed into a system of random differential equations of the form

$$\tilde{V}_{n_{\tau\tau}}(\tau,\bar{b})+\left(2k_1+\frac{\lambda}{v}\omega_n^2\right)\tilde{V}_{n_\tau}(\tau,\bar{b})+\omega_n^2(1-v^2)\tilde{V}_n(\tau,\bar{b})+\omega_n^2 d\sum_{j=1}^{N}c_j e^{b_j d\tau}\,u_{nj}(\tau,\bar{b})=\tilde{F}_n(\tau). \qquad (18)$$

The initial conditions will look like

$$\tilde{V}_n(0,\bar{b})=2\int_0^1\tilde{V}_1(\xi)\sin\left(\omega_n\xi\right)d\xi;\ \tilde{V}_{n_\tau}(0,\bar{b})=0;\ u_{nj}(0,\bar{b})=0. \qquad (19)$$

The study of the system (18) - (19) is possible using the statistical numerical Monte Carlo method [11-13].

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