On the probabilistic representation of the resolvent of the two-dimensional Laplacian

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Abstract:

Let $w(\tau) = (w_1(\tau), w_2(\tau))$, $\tau \geqslant 0$, $w(0) = (0,0)$ be a two-dimensional Wiener process. Consider a family of random linear operators

$$\mathcal{A}_\lambda^t f(x) = \int_0^t e^{\lambda\tau} f(x - w(\tau))\, d\tau, \tag{1}$$

defined on the functions $f(x) \in L_\infty \cap C(\mathbb{R}^2)$ for all $t > 0$ and $\lambda \in \mathbb{C}$, $\operatorname{Re}\lambda < 0$.

Such an operator family arises in the construction of a probabilistic representation of the resolvent of the two-dimensional Laplacian.

Namely, the following relation holds

$$(-\frac{1}{2}\Delta - \lambda I)^{-1} f(x) = \int_0^\infty e^{\lambda\tau}\, \mathbf{E}\, f(x - w(\tau))\, d\tau = (u) \lim_{t\to\infty} \mathbf{E}\, [\mathcal{A}_\lambda^t f(x)] \tag{2}$$

for all functions $f(x) \in L_\infty \cap C(\mathbb{R}^2)$.

Note that the operator $\mathcal{A}_\lambda^t$ cannot be extended to an integral operator on the entire space $L_2(\mathbb{R}^2)$. In particular, from a probabilistic point of view, this means that the process $w(\tau)$ does not have local time at an arbitrary point $x \in \mathbb{R}^2$ by time $t > 0$.

We will construct a family of random integral operators $\mathcal{R}_\lambda^t$ defined on the entire space $L_2(\mathbb{R}^2)$ and satisfying the relation

$$(-\frac{1}{2}\Delta - \lambda I)^{-1} f(x) = (L_2) \lim_{t\to\infty} \mathbf{E}\, [\mathcal{R}_\lambda^t f(x)] \tag{3}$$

for all $\lambda \in \mathbb{C}$, $\operatorname{Re}\lambda \leqslant 0$.

It will be shown that the kernels $r_\lambda(t,\cdot)$ of the corresponding operators belong with probability 1 to the Sobolev class $W_2^\alpha(\mathbb{R}^2)$, $0 \leqslant \alpha < 1/2$. Also, for the function $r_\lambda(t,\cdot)$, an explicit formula will be obtained in the form of a trajectory functional of the two-dimensional Wiener process $w(\tau)$.

References

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