On Decomposable Semiregenerative Processes and their Application to a Double Redundant Renewable System

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The talk consists from two parts. The first one deals with a short review of the Smith's regenerative idea [1] development. The notion of Decomposable Semi-Regenerative Processes (DSRP), proposed in [2] (see also [3, 4]) as its generalization, is reminded. The review of some previous its applications, which one can find in [3], is also given. In the second part of the talk some new application the DSRP theory is proposed.

1. The model. Notations and Assumptions

Consider a homogeneous repairable double redundant system with arbitrary distributed life and repair times of its components. For the reparable system, at least two different disciplines of its repair are possible: the partial and the full repair discipline. Denote by A_i, B_i, C_i ($i = 1, 2, \dots$) the life-, partial and full repair times of the system units and a whole system after their failures. Suppose that all these random variables (r.v.'s) are mutually independent and identically distributed (i.i.d.). Thus, denote by $A(t) = \mathbb{P}\{A_i \le t\}$, $B(t) = \mathbb{P}\{B_i \le t\}$ and $C(t) = \mathbb{P}\{C_i \le t\}$ the corresponding cumulative distribution functions (c.d.f.'s). Suppose that the instantaneous failures and repairs are impossible $A(0) = B(0) = C(0) = 0$, and their mean times are finite:

$$a = \mathbb{E}[A_i] < \infty, \quad b = \mathbb{E}[B_i] < \infty, \quad c = \mathbb{E}[C_i] < \infty.$$

Denote by $E = \{i = 0, 1, 2\}$ the set of system states, where i stands for the number of failed units, and introduce a random process $J = \{J(t),\, t \ge 0\}$, where

$$J(t) = \{\text{number of failed units at time } t.\}.$$

In the talk the reliability function $R(t)$, time-dependent system state probabilities (t.d.s.p.'s) $\pi_j(t) = \mathbb{P}\{J(t) = j]\}$ ($j = 0, 1, 2$), and the steady state probabilities (s.s.p.'s) $\pi_i = \lim\limits_{t\to\infty} \pi_j(t)$ ($j = 0, 1, 2$), are studied.

2. The Modified Moment Generation Functions

The following notations will be used further.

- The moment generation functions (m.g.f.'s) of the r.v.'s are:

$$\tilde{a}(s) = \mathbb{E}\left[e^{-sA}\right] = \int\limits_0^\infty e^{-sx} dA(x), \quad \tilde{b}(s) = \mathbb{E}\left[e^{-sB}\right] = \int\limits_0^\infty e^{-sx} dB(x),$$

- The modified moment generation functions (m.m.g.f.'s) are:

$$\tilde{a}_B(s) = \int_0^\infty e^{-sx} B(x) dA(x), \qquad \tilde{b}_A(s) = \int_0^\infty e^{-sx} A(x) dB(x). \quad (1)$$

- Corresponding truncated expectations are:

$$a_B = \int_0^\infty x B(x) dA(x), \quad b_A = \int_0^\infty x A(x) dB(x). \quad (2)$$

- The probabilities $\mathbb{P}\{B \le A\}$ and $\mathbb{P}\{B \ge A\}$ are associated with m.m.g.f.'s as:

$$\tilde{a}_B(0) = \mathbb{P}\{B \le A\} \equiv p, \quad \tilde{b}_A(0) = \mathbb{P}\{B > A\} \equiv q = 1 - p.$$

- Note the property of transformations (1):

$$\tilde{a}_{1-B}(s) = \tilde{a}(s) - \tilde{a}_B(s), \quad \tilde{b}_{1-A}(s) = \tilde{b}(s) - \tilde{b}_A(s). \quad (3)$$

3. Main results

The Laplace Transform (LT) $\tilde{R}(s)$ of the system reliability function $R(t)$ has been found in terms of m.g.f.'s and m.m.d.f.'s of the initial r.v.'s and contains in the following theorem [5].

Theorem 1. *The LT $\tilde{R}(s)$ of the system reliability function $R(t)$ does not depend on the repair disciplines and has the form*

$$\tilde{R}(s) = \frac{(1 - \tilde{a}(s))(1 + \tilde{a}(s) - \tilde{a}_B(s))}{s(1 - \tilde{a}_B(s))}. \quad (4)$$

From the theorem it follows that the mean system lifetime is $\tilde{R}(0) = \frac{a}{q}$.

The LTs of the t.d.s.p.'s has been obtained in [6], for the system under partial repair discipline and in [7] for the system under full repair discipline. However, they have rather complex expressions, and will be represented in the full talk, but are omitted here.

The appropriate s.s.p.'s for the system under partial repair discipline has been found in [7] and are given in the following theorem

Theorem 2. *The s.s.p.'s for the system under partial repair discipline are:*

$$\pi_0 = \quad 1 - \tfrac{b}{a_B + b_A}, \quad \pi_1 = \tfrac{a+b}{a_B+b_A} - 1, \quad \pi_2 = 1 - \tfrac{a}{a_B+b_A}. \tag{5}$$

At least for the system under full repair discipline the following theorem holds

Theorem 3. *The s.s.p.'s for the system under full repair discipline are:*

$$\pi_0 = \frac{aq + a_B + b_A - b}{a + q(a+c)}, \quad \pi_1 = \frac{a + b - (a_B + b_A)}{a + q(a+c)}, \quad \pi_2 = \frac{cq}{a + q(a+c)}. \tag{6}$$

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