Tight asymptotic of probability of singularity of Bernulli square matrices

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There is a number of works devoted to determination of tight asymptotic of probability P_n that random square matrix with independent uniformly distributed ± 1 entries is singular. In this paper we prove the general

Conjecture 1 *The following asymptotic equality is valid*

$$P_n = n^2 2^{1-n}(1+o(1)). \tag{1}$$

Using the same arguments as in the following proof of this conjecture we prove the following asymptotic equality is valid

$$P_n - 4\binom{n}{2}2^{-n} \sim 16\binom{n}{4}\left(\frac{3}{8}\right)^n (1+o(1)). \tag{2}$$

History of the problem.

The history of the problem of determining upper bound for P_n started in 1967 when Komlós proved that $P_n = o(1)$. In 1995 Kahn, Koml'os and Szemeredi proved that $P_n < (\alpha + o(1))^n$ for some $\alpha < 1$ very closed to 1. Actually that work established many interesting ideas which also used later in improvements of this bound. First such improvement was made by Tao and Vu, who improve α to 0.939 and in later work to 0.75. Their improvement add additive combinatorics as ingredient in the proof and needs better estimation of $|\cos\phi|$. This last bound was improved by Bourgain, Wood and Vu to

$$P_n \le \left(\frac{1}{2} + o(1)\right)^{n/2}. \tag{3}$$

K.Tikhomirov proved tight logarithmic relation

$$P_n \le \left(\frac{1}{2} + o(1)\right)^{n}. \tag{4}$$