Gliklikh Yu.E. (Voronezh State University, Voronezh, Russia) On a certain second order stochastic algebraic-differential equation with mean derivatives¹

The preliminaries and notation can be found in [1].

We investigate the stochastic algebraic-differential equation

$$\frac{1}{2}(DD_* + D_*D)L(\eta(t) + \int_0^t w(s)ds) = M\eta(t) + f(t) + D_Sw(t)$$
(1)

where L is a degenerate matrix, M is a non-degenerate matrix such that the matrix pencil $\lambda L + M$ is non-degenerate, f(t) is a deterministic smooth vector function and $D_S w(t)$ is the symmetric mean derivative (current velocity) of a Wiener process. The matrices L and M together describe a certian electronic device, f(t) is the incoming into the device signal, $\eta(t)$ is the outgoing signal, $\int_0^t w(s) ds$ and $D_S w(t)$ are noises. The second derivative $(DD_* + D_*D)$ was introduced by Edward Nelson in his Stochastic Mechanics (a version of quantum mechanics). We interpret a solution of (1) as the outgoing signal that takes into account the noise in incoming signal and the noise generated by quantum-mechanical processes in the device.

Since in (1) besides the forward mean derivative D there are the backward mean derivative D_* and the symmetric mean derivative D_S , (1) is ill-defined at t = 0. Select an arbitrary small value $t_0 > 0$.

Theorem. There exists a process $\eta(t)$, well defined for $t \ge 0$, that satisfies (1) for $t \ge t_0$.

References

1. Gliklikh Yu.E. Global and stochastic analysis with applications to mathematical physics.- London: Springer-Verlag,2011.- 460 p.

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