Fixed-width confidence interval estimation of functionals of an unknown distribution function.

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Consider a sequence of independent, identically distributed random variables $\xi_1, \xi_2, ..., \xi_n, ...$ with a distribution function $F(x)$. In order to estimate functional $\theta(F)$ of this distribution function $F(x)$ we consider a statistical estimators $\theta_n(F) = \theta_n(\xi_1, \xi_2, ..., \xi_n)$, which have finite expectations and, therefore, can be represented in the following form, $\theta_n(F) = \theta(F) + \sum_{k=1}^{n} Y_n(F, \xi_k) + Z_n$, where $Y_n(F, \xi_k), 1 \le k \le n$ и $Z_n = E(\theta_n(F)) - \theta(F)$, which satisfy the following condition: $n^\alpha \sum_{k=1}^{n} Y_n(F, \xi_k) \xrightarrow{D} N(0, \sigma^2(F))$ and $n^\alpha Z_n \xrightarrow{P} 0$ as $n \to \infty$, here $0 < \sigma^2(F) < \infty$, $\alpha > 0$.

Let be $0 < \gamma < 1$, $a = \Phi^{-1}\left((1+\gamma)/2\right)$, $\Phi(y) = (2\pi)^{-1/2} \int_{-\infty}^{y} e^{-t^2/2} dt$ and $V_n^2 = V_n^2(\xi_1, \xi_2, ..., \xi_2)$ consistent estimators of $\sigma^2(F)$. Introduce the stopping time

$$N_\varepsilon = \inf\left(n \ge 1 : n \ge \left(a^2 V_n^2 / \varepsilon^2 \right)^{(2\alpha)^{-1}} \right), \qquad \varepsilon > 0 .$$

Theorem. 1) If $V_n^2 \xrightarrow{P} \sigma^2(F)$ as $n \to \infty$, then $N_\varepsilon / n_\varepsilon \xrightarrow{P} 1$ as $\varepsilon \to 0$.

2) If $n^\alpha \left(E(\theta_n(F)) - \theta(F) \right) \to 0$ as $n \to \infty$ and $\zeta_\varepsilon(t), t \in [0,1] \xrightarrow{J} W(t), t \in [0,1]$ as $\varepsilon \to 0$, here $\zeta_\varepsilon(t) = \sigma^{-1}(F) \cdot n_\varepsilon^\alpha \cdot \sum_{k=1}^{[n_\varepsilon t]} Y_{n_\varepsilon}(F, \xi_k), t \in [0,1]$, $W(t), t \in [0,1]$ is the Wiener process, symbol $\xrightarrow{J}$ denotes convergence in Skorokhod J - topology, then $\sigma^{-1}(F) N_\varepsilon^\alpha \left(\theta_{N_\varepsilon}(F) - \theta(F) \right) \xrightarrow{D} N(0,1)$ as $\varepsilon \to 0$ and $\lim_{\varepsilon \to 0} P\left(\theta\left(F\right) \in I(N_\varepsilon) \right) \ge \gamma$.

References

1. Rakhimova G. G.,"Application of limit theorems for superposition of random functions to sequential estimation", In the book: Silvestrov S., Rancic M., Malyarenko A. (eds.) Stochastic Processes and Applications. Chapter 7, p.148-154. Springer, Proceedings in Mathematics & Statistics, 271, Springer, Cham, 2018.