Shumafov M. M., Tlyachev V. B., Panesh T. A. (Adyghe State University, Republic of Adygea, Russia) — On the stochastic stability of solutions of the second order differential equations perturbed by white noise.

In the present work, we consider the second order differential equations which are typical in the theory of nonlinear dynamics and for these ones we prove stochastic stability theorems that generalize the results of papers [1], [2].

Let us formulate one of these theorems, concerning the generalized Rayleigh equation perturbed by white noise:

$$\ddot{x} + f(x)\varphi(\dot{x}) + g(x) = \sigma(\dot{x})\dot{\xi}(t),$$

where the functions f(x), $\varphi(y)$, g(x) and $\sigma(y)$ satisfy the Lipschitz condition on \mathbb{R} , and $\dot{\xi}(t)$ is white noise. Assume that $\varphi(0) = g(0) = \sigma(0) = 0$.

Theorem. Suppose there exist numbers $b_1 > 0$, $b_2 > 0$ and σ_0 such that the following hypotheses are fulfilled:

1) $f(x) \ge b_1$ for all x,

2) $\varphi(y)/y \ge b_2$ for all $y \ne 0$,

3) xg(x) > 0 for all $x \neq 0$ and $\int_{0}^{x} g(s)ds \rightarrow +\infty$ as $|x| \rightarrow +\infty$,

4) $0 < \sigma(y)/y < \sigma_0^2$ for all $y \neq 0$ and $\sigma_0^2 \leq 2b_1b_2$.

Then the solution $(x(t) \equiv 0, y(t) \equiv 0)$ of the stochastic Rayleigh system in the form of Ito

$$dx = ydt, \ \ dy = -[f(x)\varphi(y) + g(x)]dt + \sigma(y)d\xi(t)$$

is globally asymptotically stable in probability.

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