V. I. Afanasyev. Limit theorems for functionals of a branching process in random environment starting with a large number of particles[1]

Let $\left\{Z_i^{(k)}, i \in \mathbf{N}_0\right\}$ be a branching process in a random environment $\{Q_i, i \in \mathbf{N}\}$, starting with k particles. Let $f_i(\cdot)$ be the generating distribution function Q_i. We set $X_i = \ln f_i'(1)$, $\eta_i = f_i''(1) / (f_i'(1))^2$ and introduce the associated random walk $S_0 = 0$, $S_n = \sum_{i=1}^n X_i$, $n \in \mathbf{N}$.

Assumption A. The random variable X_1 belongs without centering to the domain of attraction of some strictly stable law with index $\in (0,2]$ and $((0,+\infty)) \in (0,1)$.

Assumption B. For some $q > 0$ $\mathbf{E} \ln^{\alpha+q}(\eta_1 \vee 1) < +\infty$.

We set $U_n = \left\{S_{\lfloor nt \rfloor}, t \geq 0\right\}$. By Skorokhod's theorem, there exists such a positive numerical sequence $\{A_n, n \in \mathbf{N}\}$ that, as $n \to \infty$, $U_n / A_n \xrightarrow{D} U$, where $U = \{U(t), t \geq 0\}$ is a strictly stable Levy process with index $\alpha \in (0,2]$.

We fix $x \in (0,+\infty)$. Suppose that $k = k_n$ and $\ln k_n \sim A_n x$ for $n \to \infty$. Let $T^{(k_n)}$ be the extinction time of the branching process $\left\{Z_i^{(k_n)}, i \in \mathbf{N}_0\right\}$ and $M^{(k_n)} = \max_{0 \leq i < +\infty} Z_i^{(k_n)}$, $\Sigma^{(k_n)} = \sum_{i=0}^{\infty} Z_i^{(k_n)}$.

We set $L(t) = \inf_{0 \leq s \leq t} U(s)$, $M(t) = \sup_{0 \leq s \leq t} U(s)$ for $t \geq 0$ and $M_x = x + M(T_{-x})$. Let T_{-a} be the time of the first attaining of the set $(-\infty, -a]$, where $a > 0$, by the process $\{L(t), t \geq 0\}$.

Theorem. If the assumptions A, B are fulfilled, then, as $n \to \infty$,

$$\frac{T^{(k_n)}}{n} \xrightarrow{D} T_{-x}, \qquad \frac{\ln M^{(k_n)}}{A_n} \xrightarrow{D} M_x, \qquad \frac{\ln \Sigma^{(k_n)}}{A_n} \xrightarrow{D} M_x.$$

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