Review of certain topical directions of stochastic analysis and limit theorems

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A review of a recent progress in the areas of limit theorems and stochastic analysis will be offered. Here we only show one LLN and one SDE theorem.

1. Consider a sequence of r.v.'s (ξ_n) uniformly integrable in Césaro sense.

Theorem 1 (Extension of Y.S. Chow LLN result 1971) *Let* $\mathsf{E}\xi_n \stackrel{\forall n}{=} 0$ *and*

$$\frac{1}{n}\sum_{k=1}^{n} \mathsf{E}|\mathsf{E}(\xi_k|\xi_1+\ldots+\xi_{k-1})| \to 0, \quad n\to\infty.$$

Then

$$\frac{1}{n}\sum_{k=1}^{n} \xi_k \stackrel{\mathsf{P}}{\to} 0, \quad n\to\infty.$$

2. Consider an SDE in $\mathbb{R}^d$, $d\ge 1$ (here W_t is a Wiener process in $\mathbb{R}^d$)

$$dX_t = \sigma(X_t)dW_t + b(X_t)dt, \quad X_0 = x, \tag{1}$$

with a nondegenerate **diagonal** matrix σ of the form $\sigma_{ii}(x^i)$; all σ_{ii} are Hölder-1/2. Let the drift b have a form, $b^i(x) = b_0^i(x^i) + b_1^i(x)$, where b_1 is Lipschitz, while b_0 is only Borel measurable; all coefficients are bounded. The next theorem (and a more general one) is obtained jointly with Anastasiia Lyappieva (MSU).

Theorem 2 (Extension of T. Yamada & S. Watanabe result 1971) *The equation (1) has a pathwise unique strong solution.*

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