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ABSTRACTS OF THE INTERNATIONAL CONFERENCE ON STOCHASTIC METHODS

(*Translated by D. V. Kropotova*)

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The First International Conference on Stochastic Methods was held May 27–June 3, 2016, in the village of Abrau-Durso (at the Moryak Hotel) on the Black Sea coast. This conference was a continuation of the All-Russian School-Colloquium on Stochastic Methods regularly held (20 times) in different places in the Russian Federation. The organizers of this conference were Steklov Mathematical Institute RAS (Department of Theory of Probability and Mathematical Statistics), Lomonosov Moscow State University (Department of Probability Theory), and Don State Technical University (Department of Higher Mathematics) (Rostov-on-Don). The chairman of this conference was academician of the RAS A. N. Shiryaev. The conference committees were as follows. Organizing Committee: I. V. Pavlov (Deputy Chairman), E. V. Burnaev, M. V. Zhitlukhin, V. V. Shamraeva, and S. Ya. Shatskikh; the Program Committee: P. A. Yaskov (Deputy Chairman), Yu. E. Gliklikh, S. B. Klimentov, V. V. Ulyanov; the Publishing Committee: T. B. Tolozova, E. B. Yarovaya. Technical problems at the conference were considered by the Local Committee: I. V. Pavlov (Chairman), V. V. Shamraeva (Deputy Chairman), S. I. Uglich.

In addition to scientists from Russia, scientists from the USA and Uzbekistan took part in the conference. Eleven 40-minute keynote talks, 20 section talks, and 14 poster presentations were given. The themes of the keynote talks were the following: A. N. Shiryaev, *Randomness in probability*; A. A. Lykov (jointly with V. A. Malyshev and M. V. Melikyan), *New applications of stochastic methods in physics*, P. A. Yaskov, *On necessary and sufficient conditions in the Marchenko–Pastur theorem on spectral distributions of random matrices*; E. B. Yarovaya, *Stochastic evolution of a particle system in a noncompact phase space: An approach focused on branching random walks*; V. V. Ulyanov, *On a common approach to estimates of approximation exactness*; O. E. Kudryavtsev, *New approaches to the calculation of the prices of exotic options in Lévy models*; M. V. Zhitlukhin, K. A. Borovkov, Yu. S. Mishura (jointly with A. A. Novikov), *On maxima of Gaussian processes and their approximations*; Ya. I. Belopolskaya, *Stochastic models of conservation laws in physics and biology*; Yu. E. Gliklikh, *Description of the motion of a quantum particle in the classic gauge field in the language of stochastic mechanics*; S. Ya. Shatskikh (jointly with L. E. Melkumova), *Geometry of conditional quantiles of multidimensional probability distributions*; I. V. Pavlov, *Stochastic analysis on deformed structures: Survey of results and main directions for further research.*

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The Second International Conference on Stochastic Methods will held at the end of May 2017 at the same location.

A. N. Shiryaev, I. V. Pavlov, T. B. Tolozova, V. V. Shamraeva

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S. Zh. Aibatov, L. G. Afanasyeva (Moscow, Russia) — **Subexponentiality conditions of stationary waiting time in a single-server queue with regenerative input flow.**

A single-server queue with a regenerative input flow and random service time of the request is considered. The input flow $X(t)$ is a total task received by the system during the time period $(0,t]$. The sequence $\{\theta_n\}_{n=1}^{\infty}$ $(\theta_0 = 0)$ is the regeneration points of $X(t)$. Denote $\tau_n = \theta_n - \theta_{n-1}$, $\gamma_n = X(\theta_n) - X(\theta_{n-1})$. Assume that $\mathbb{E}\,\tau_n < \infty$ and $\mathbb{E}\,\gamma_n < \infty$. Let $G(y) = \mathrm{Prob}\,(\gamma_n \leqslant x)$, $\overline{G}(y) = 1 - G(y)$, $G^I(x) = (\mathbb{E}\,\gamma_n)^{-1}\int_0^x \overline{G}(y)\,dy$.

Let us introduce a process of virtual waiting time $W(t)$ and embedded processes $W_n = W(\theta_n - 0)$, $w_n = W(t_n - 0)$, where t_n is a moment of the nth jump of the process $X(t)$. Assume that the traffic coefficient of the system $\rho = \mathbb{E}\,\gamma_n/\mathbb{E}\,\tau_n < 1$; then processes $W(t)$, W_n, w_n have limit stationary distributions $\Psi(x)$, $\Phi(x)$, $F(x)$, respectively.

THEOREM 1. *Let $G^I(x)$ be a subexponential distribution function and* $\mathrm{Prob}\,(\gamma_n > y + \tau_n) \sim \overline{G}(y)$ *as $y \to \infty$; then as $x \to \infty$*

$$1 - \Phi(x) \sim \frac{\rho}{1-\rho} G^I(x). \tag{1}$$

Under some additional assumptions the asymptotic (1) is true for functions $\Psi(x)$ and $F(x)$. We have applied Theorem 1 to the classical system $\mathrm{Reg}/G/1/\infty$, where it is assumed that the service times are independent identically distributed random variables which do not depend on the input flow $A(t)$. ($A(t)$ is the number of claims received by the system during $(0,t]$.)

Yu. V. Averbuch (Ekaterinburg, Russia) — **Approximate solutions of stochastic continuous-time games.**[1]

In this talk, based on [1], we consider a stochastic controlled continuous-time system. In the initial problem a near-optimal strategy is constructed on the solution of the control problem for a modeling system. It is assumed that this solution is known. The dynamics of each system is specified by a Lévy–Khintchine-type generator. It is also assumed that each system is controlled by two players with opposite objectives. For simplicity, we assume that the purpose of the first (respectively, second) player is to minimize (respectively, maximize) variables $\mathbf{E}\,g(X(T))$.

We introduce a notion of a u-stable function for the modeling system. The main result is as follows: If c_+ is a u-stable function for the modeling system, then the upper price function in the initial game $\mathrm{Val}_+(s,y)$ does not exceed $c_+(s,y) + R\cdot C\sqrt{\varkappa + M_0^1 + M_0^2}$. Herein R is a Lipschitz constant for the payoff function g; C is determined by the Lipschitz constants for the dynamics functions; $\varkappa$ gives "the distance" between the initial and modeling systems; M_0^i, $i = 1,2$, describe the degree "of stochasticity" of the initial and modeling systems. Based on this statement, we can construct a near-optimal control in a system with a large number of particles with a finite number of states that approximates the optimal control in the limiting deterministic system. Independently, this result was obtained in [2].

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Ya. I. Belopolskaya (St. Petersburg, Russia) — **Stochastic models of conservation laws in physics and biology.**[2]

Our talk is devoted to the establishment of connections between systems of nonlinear parabolic equations, arising as mathematical models of various phenomena in physics,

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chemistry, biology, and other fields, and the theory of stochastic equations. We discuss the probabilistic interpretation of classical and generalized solutions of the Cauchy problem for systems of nonlinear parabolic equations of two types: (1) with diagonal entry of highest order terms (and, in particular, for parabolic perturbations of hyperbolic conservation laws and balance); (2) for systems with nondiagonal entry of highest order terms (parabolic systems with cross-diffusion). Special attention is paid to the problem of constructing probabilistic representations of generalized solutions of the Cauchy problem for systems of both types.

A common approach to the construction of a probability representation (classical, generalized, or viscous) of the solution of the Cauchy problem for nonlinear parabolic equations and systems can be divided into three steps. In the first step, we construct a probability representation for the desired solution of the Cauchy problem, assuming that such a solution exists and is unique and twice differentiable. In the second step, we construct a closed system of stochastic relations, including the probability representation obtained in the first step, and assuming that the solution of this stochastic system, which has the required properties, exists, we need to verify that as a result we have simultaneously constructed the required solution of the Cauchy problem for the initial system of parabolic equations. Finally, in the third step, rejecting any a priori assumptions, it is necessary to investigate the closed stochastic system obtained in the second step to prove the existence and uniqueness of its solution and to verify whether this solution has the required properties or not.

Note that this approach is effective in the construction of classical and generalized solutions and in the construction of viscous solutions for systems of type (1). For these systems all three of the above-described steps of constructing classical [1], [2], generalized [3], and viscous [4] solutions of the Cauchy problem were realized. For systems of type (2) only the first two steps of the above-described approach to the construction of a generalized solution of the Cauchy problem (see [5]–[7]) were realized.

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E. V. Burnaev and A. A. Artemov (Moscow, Russia) — **Allocation of trend from long-memory noise and detection of disorders in the background.**[3]

We consider the problem of estimating the parameter $\boldsymbol{\theta} = (\theta_1, \dots, \theta_n)$ of the drift coefficient of fractal Brownian motion, which has the form $\sum_{i=1}^{n} \theta_i \varphi_I(t)$, where $\varphi_i(t)$, $i = 1, \dots, n$, is a set of known functions. For $\boldsymbol{\theta}$ the maximum likelihood estimate is obtained, as well are Bayesian estimates for normal and uniform a priori distributions (see [1]). On the basis of the obtained estimates, an algorithm for allocating the quasi-periodic trend and detecting disorders in the background has been developed. The definition of the moment

[3]This work was supported by IITP RAS and RSF grant 14-50-00150.

time of the "alarm" in the form $\tau = \inf\{t \geqslant 0\colon a_t \geqslant h\}$, where $a_t = \psi(\boldsymbol{\lambda}, \mathbf{S}_t^1, \dots, \mathbf{S}_t^d)$, $\boldsymbol{\lambda}$ are the adjusting parameters of the aggregation function $\psi(\cdot)$, $\mathbf{S}_t^k = \{s_s^k, 0 \leqslant s \leqslant t\}$, $k = 1, \dots, d$, are trajectories of the standard statistics $\{s_t^k\}_{k=1}^d$ for the disorder detection, permitted us to increase significantly the accuracy of the disorder detection (see [2]). Application of the developed algorithm is illustrated by the example of solving the problem of predictive maintenance of software-loaded systems (see [3]).

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E. A. Chernavskaya and E. E. Bashtova (Moscow, Russia) — **Limit theorems for an infinite-server queuing system and regenerating input flow.**[4]

We consider an infinite-server queuing system with a regenerating input flow $X(t)$, with regeneration points $\{\theta_i, i \geqslant 0\}$, $\theta_0 = 0$. This system is a generalization of the system in [1]. Denote $\xi_i = X(\theta_i) - X(\theta_{i-1})$, $\tau_i = \theta_i - \theta_{i-1}$, $i \geqslant 1$. Let the service times be a sequence of independent identically distributed random variables with the distribution function $B(t)$. We assume that $1 - B(t) \sim \mathcal{L}(t)t^\beta$ for $t \to \infty$, where $0 < \beta < 1$ and $\mathcal{L}(t)$ is a slowly varying function. Let $q(t)$ be the number of requests served in the system at time t. Denote $\kappa = (1-\beta)^{-1}\mathbf{E}\,\xi_1/\mathbf{E}\,\tau_1$.

Theorem. *Let* $\mathbf{E}\,\tau_1^r < \infty$, $r > 2$, $\mathbf{E}\,\xi_1^2 < \infty$; *then*

$$\frac{q(t) - \kappa t^{1-\beta}\mathcal{L}(t)}{\sqrt{t^{1-\beta}\mathcal{L}(t)}} \xrightarrow{d} \mathcal{N}(0, \kappa) \quad \textit{for } t \to \infty.$$

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Yu. E. Gliklikh (Voronezh, Russia) — **Description of the motion of a quantum particle in the classic gauge field in the language of stochastic mechanics.**[5]

This talk is devoted to the development of some of the results of [1], [2], [3], [4], [5], [6], and [7]. Nelson's stochastic mechanics is a mathematical theory based on classical physics, but it gives the same predictions as quantum mechanics for a wide class of problems where both theories are applicable. We can assume that stochastic mechanics is a special method of quantization different from the Hamiltonian and Lagrangian (in terms of integrals) methods. One of the main distinguishing features of stochastic mechanics is that it is the quantized Newton's second law, not the Hamilton or Lagrange equations. The stochastic analogue of Newton's law is known as the Newton–Nelson equation.

The main notion used to describe the Newton–Nelson equations is the so-called mean derivatives of stochastic processes, introduced by Nelson, as well as the quadratic mean derivative introduced in [1] (see also [2] and [3]). Note that equations with mean derivatives are now used also in other models in physics, economics, etc.

At present, a large number of problems of quantum theory, both ordinary and relativistic (in the relativistic case the definition of mean derivatives had to be modified, since it turned out that the classical derivatives are noncovariant with respect to the Lorentz transformation)

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were investigated in the language of stochastic mechanics. However, there was no description of the motion of a quantum particle in a gauge field, apparently due to the fact that previously there was no description of the classical particle in a gauge field in terms of Newton's second law. This description was proposed in [4] (see also [2] and [3]), which proposed a special second order equation for a bundle with connection, which was interpreted as Newton's second law describing the motion of a classical particle in a classical gauge field. Basing on this investigation we study the appropriate Newton–Nelson equation in bundles with connections. We consider two cases: when the bundle base is the Riemann manifold and the bundle itself (main and vector) is real, and when the bundle base is a space–time general theory relativity and the bundle is complex. The latter case is interpreted as a description of motion of a relativistic quantum particle in a classical gauge field. For a particular case of the symmetry group $U(1)$ we analyze the connection with quantum electrodynamics. Preliminary versions of these results were published in [5], [6], and [7].

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S. D. Gorban (Moscow, Russia) — **The sequential testing problem on a finite horizon for fractal Brownian motion.**

Let the observed process $(X_t)_{t\geq 0}$ be such that $X_t = \theta\mu t + \sigma B_t^H$, where $(B_t^H)_{t\geq 0}$ is a fractal Brownian motion with the Hurst exponent H, $\mu \neq 0$, $\sigma^2 > 0$, and θ is a random variable independent of B^H and taking two values $\mathbf{P}(\theta = 1) = \pi$ and $\mathbf{P}(\theta = 0) = (1 - \pi)$. The sequential testing problem is to find the optimal decision rule (τ, d), where $0 \leqslant \tau \leqslant T$ is a stopping time with respect to the natural filtration of the process X, and d is an $\mathcal{F}_\tau^X$-measurable random variable taking values 0 and 1:

$$(\tau^*, d^*) = \underset{(\tau,d)}{\arg\min}\, \mathbf{E}_\pi \left[\tau + aI(d = 0,\ \theta = 1) + bI(d = 1,\ \theta = 0)\right].$$

With the help of integral transformation, which was introduced by A. A. Muravlev in [2], and time substitution it is possible to reduce the problem for fractal Brownian motion to the corresponding problem for the Wiener process with nonlinear penalty on inquiry. We show that sequential hypotheses testing is reduced to the optimal stopping problem and to the associated free boundary problem with. Similarly [1], it is established that the continuation set decreases "sharply" at the terminal time moment.

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A. S. Grechko and O. E. Kudryavtsev (Rostov-on-Don, Russia) — **Features of construction of the index of volatility of the Russian derivatives market, taking into account possible price jumps.**[6]

Recently, the volatility of the Russian currency and stock markets has increased significantly; as a result, the need for risk hedging with the help of the derivatives market instruments of the Moscow stock exchange increased. There was a need for a Russian volatility index RVI, an analogue of the U.S. index VIX.

The analysis carried out by the authors of [1] shows that the existing indices based on the free-model volatility formula poorly estimate the realized volatility in the case of the Russian market.

This technique can be applied for diffusion processes and processes with rare jumps, but Lévy processes with unbounded variation — for example, the well-known CGMY model — are most suitable. The authors obtained the free-model volatility formula for Lévy processes, which was tested for real data from the Russian derivatives market. Thus, we obtained a new methodology for calculating the volatility index, taking into account the jumps in the dynamics of the RTS index.

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N. V. Gribkova (St. Petersburg, Russia) — **Probabilities of large and moderate deviations of truncated *L*-statistics.**

Let $X_1, X_2, \dots$ be a sequence of independent identically distributed random variables, $X_i \in \mathbf{R}$, with a distribution function F; $X_{1:n} \leqslant \cdots \leqslant X_{n:n}$ be order statistics corresponding to the first n observations; $F^{-1}(u) = \inf\{x\colon F(x) \geqslant u\}$, $0 < u < 1$; F_n and F_n^{-1} be the empirical distribution function and its inversion function, respectively. We consider a (weakly) truncated mean

$$T_n = \frac{1}{n}\sum_{i=k_n+1}^{n-m_n} X_{i:n} = \int_{\alpha_n}^{1-\beta_n} F_n^{-1}(u)\,du,$$

and let

$$\mu_n = \int_{\alpha_n}^{1-\beta_n} F^{-1}(u)\,du,$$

where k_n, m_n are integers, $0 \leqslant k_n < n - m_n \leqslant n$, $k_n \wedge m_n \to \infty$, $n \to \infty$, $\alpha_n = k_n/n$, $\beta_n = m_n/n$. Let $\xi_\nu = F^{-1}(\nu)$, $0 < \nu < 1$, introduce Winsorized random variables $W_i^{(n)} = \xi_{\alpha_n} \vee (X_i \wedge \xi_{1-\beta_n})$. Denote $\sigma_{W,n} = (\mathbf{D}(W_i^{(n)}))^{1/2}$ and assume that $\liminf \sigma_{W,n} > 0$.

One of the main results of our topic is the following theorem on moderate deviations for weakly truncated sums.

Theorem. *Assume that* $\mathbf{E}\,|X_1|^p < \infty$ *for some* $p > c^2 + 2$ $(c > 0)$, *and assume also that* $\log n/(k_n \wedge m_n) \to 0$ *and* $\alpha_n \vee \beta_n = o((\log n)^{-2p/(p-2)})$. *Then*

$$\mathbf{P}\biggl(\frac{\sqrt{n}(T_n - \mu_n)}{\sigma_{W,n}} > x\biggr) = [1 - \Phi(x)](1 + o(1)),$$
$$\mathbf{P}\biggl(\frac{\sqrt{n}(T_n - \mu_n)}{\sigma_{W,n}} < -x\biggr) = \Phi(-x)(1 + o(1))$$

[6] This work was supported by RHSF grant 15-32-01390.

uniformly in x in $-A \leqslant x \leqslant c\sqrt{\log n}$ $(A > 0)$.

The proof is based on the approach proposed in [1].

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Yu. V. Gusak (Moscow, Russia) — **Stability of the solution of the optimization problem in one insurance model.**

The discrete-time model of an insurance company is considered. It is assumed that the annual claims form a sequence of independent identically distributed nonnegative random variables $\{X_n\}_{n\geqslant 1}$ such that $X_n \sim \operatorname{law}(X)$, $n \geqslant 1$. In order to avoid ruin, additional capital injections and reinsurance are made. According to the reinsurance contract, the level of one's own retention for the current year is determined at the beginning of the year. Additional injections are made at the end of the year if the company's capital falls below a fixed level a. Reinsurance parameters minimize the total expected injections for n years, provided that insurance and reinsurance premiums were calculated on the average principle with a safety load (see [1]). The stability of the minimal expected injections and optimal model parameters with respect to the change in the distribution of insurance claims is estimated. Namely, if the claims $\{X_n\}_{n\geqslant 1}$ have a distribution $\operatorname{law}(Y)$ different from $\operatorname{law}(X)$, and if the quantities X and Y are close in the Kantorovich metric, we derive an estimate for $\sup_{u\geqslant a}|h_{n_X}(u) - h_{n_Y}(u)|$, where u is the initial capital of the company, and $h_{n_X}(u)$ and $h_{n_Y}(u)$ are minimal injections for $X_n \sim \operatorname{law}(X)$ and $X_n \sim \operatorname{law}(Y)$, respectively.

Because in practice the distribution of claims is usually unknown, the stability of the solution and the key characteristics of the model are studied when the theoretical distribution is replaced by an empirical distribution.

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S. B. Klimentov (Rostov-on-Don, Russia) — **Some "pathological" solutions to a Beltrami equation.**[7]

We construct an example of a bounded solution of a uniformly elliptic Beltrami equation in the unit disk D that has no nontangential limit values a.e. on $\Gamma = \partial D$, and also an example of a solution of this equation bounded in D which is not identically zero, having zero nontangential limit values a.e. on Γ. These examples show that, in the general case for the Hardy classes of solutions of a Beltrami equation (and to more general noncanonical first-order elliptic systems), the usual statement of boundary value problems used for holomorphic and generalized analytic functions is ill-posed and also makes it possible to construct examples of random (nondiffusion) processes in D with probability one going to the set of zero linear measure on Γ.

O. E. Kudryavtsev (Rostov-on-Don, Russia) — **New approaches to the calculation of exotic options prices in Lévy models.**[8]

Calculation of exotic options prices (barrier, lookback, etc.), including an estimation of the liquidity risk or output of price for a fixed level, is based on the behavior of the processes of the supremum (infimum) of the price whose characteristic functions in the case of general Lèvy models do not have formulas convenient for numerical realization.

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Let X_t be a Lévy process. Consider the Laplace–Carson transform of the characteristic functions of the supremum process $\overline{X}_t = \sup_{0\leqslant s\leqslant t} X_s$ and infimum process $\underline{X}_t = \inf_{0\leqslant s\leqslant t} X_s$: $\phi_q^+(\xi) = \mathbf{E}\,[e^{i\xi\overline{X}_T}]$ and $\phi_q^-(\xi) = \mathbf{E}\,[e^{i\xi\underline{X}_T}]$, where $T \sim \operatorname{Exp} q$. For a wide class of Lèvy processes the following theorem is proved [1].

THEOREM 1. *There are constants $\omega_- < 0$ and $\omega_+ > 0$ such that*

(a) *$\phi_q^+(\xi)$ admits an analytic continuation for $\Im\xi > \omega_-$ and can be represented in the following form: $\phi_q^+(\xi) = \exp[i\xi F^+(0) - \xi^2 \widehat{F}^+(\xi)]$, where*

$$F^+(x) = I_{(-\infty,0]}(x)(2\pi)^{-1} \int_{-\infty+i\omega_-}^{+\infty+i\omega_-} e^{ix\eta}\, \frac{\ln(q+\psi(\eta))}{\eta^2}\, d\eta; \tag{1}$$

$$\widehat{F}^+(\xi) = \int_{-\infty}^{+\infty} e^{-ix\xi} F^+(x)\, dx. \tag{2}$$

(b) *$\phi_q^-(\xi)$ admits an analytic continuation to semiplane $\Im\xi < \omega_+$ and can be represented in the following form: $\phi_q^-(\xi) = \exp[-i\xi F^-(0) - \xi^2 \widehat{F}^-(\xi)]$, where*

$$F^-(x) = \mathbf{o}_{[0,+\infty)}(x)(2\pi)^{-1} \int_{-\infty+i\omega_+}^{+\infty+i\omega_+} e^{ix\eta}\, \frac{\ln(q+\psi(\eta))}{\eta^2}\, d\eta, \tag{3}$$

$$\widehat{F}^-(\xi) = \int_{-\infty}^{+\infty} e^{-ix\xi}\, F^-(x)\, dx. \tag{4}$$

Formulas (1)–(4) can be efficiently realized numerically using a fast Fourier transform.

Applying the Gaver–Stehfest algorithm to the function $\phi_q^+(\xi)$, we obtain the characteristic function of the random variable $\overline{X}_T$. The distribution function of the value $\overline{X}_T$ can be expressed in terms of the Fourier integral, whose value can be calculated using the fast Fourier transform. The inverse distribution function can be obtained by linear interpolation and used to construct Monte-Carlo methods that simulate the joint distribution of the maximum and the values of the Lévy process at a fixed time with the Wiener–Hopf factorization. A similar result can be obtained for the infimum processes.

On the other hand, formulas (1)–(4) can be used to solve problems of calculating the arbitrage-free prices of exotic options by the Wiener–Hopf method (see [2]) for more accurate approximation of the factors.

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D. I. Lisovskii and M. V. Zhitlukhin (Moscow, Russia) — **Disorder problem for the Brownian bridge.**

We consider the problem of early detection of the change in drift for the Brownian bridge process in the Bayesian setting. We observe a random process $X = (X_t)_{0\leqslant t\leqslant 1}$, given by the SDE

$$dX_t^\theta = \frac{\mu I_{\{t\geqslant\theta\}} - X_t^\theta}{1-t}\, dt + dB_t,$$

where $\mu > 0$ is a known numerical parameter, $B = (B_t)_{0\leqslant t\leqslant 1}$ is a standard Brownian motion, and θ is a random variable uniformly distributed on the interval $[0,1]$ (the unobserved moment of the disorder). The following risk function is considered: $\inf_{\tau\in\mathfrak{M}} \mathbf{E}^{\theta}(I_{\{\tau<\theta\}} + c(\tau-\theta)^{+})$, where the constant $c > 0$ (payment for observations), and $\mathfrak{M}$ is the class of stopping times with respect to the filtration $(\mathcal{F}_t^{X^\theta})_{0\leqslant t\leqslant 1}$. Here, the averaging is performed with respect to the special constructed measure Prob^{θ} (see [1]). We have to prove that the formulated problem of debugging is equivalent to the problem of optimal stopping $\inf_{\tau\in\mathfrak{M}} \mathbf{E}^{1}(1-\tau+c\int_0^{\tau}\psi_s\,ds)$, where the process ψ_s is a Shiryaev–Roberts statistic, more precisely, $d\psi_t = dt + \psi_t(\mu/(1-t))\,dB_t$, $\psi_0 = 0$ for any $t \in [0,1)$. Infimums in these problems are achieved at the same stopping time. To solve the optimal stopping problem, we apply the methods described in [2]; in particular, both numerical methods and Monte-Carlo methods are used.

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A. A. Lykov, V. A. Malyshev, and M. V. Melikyan (Moscow, Russia) — **New applications of stochastic methods in physics.**

The second half of the previous century was characterized by fantastic development of probabilistic methods in equilibrium statistical physics. This presentation is devoted to new rigorous mathematical results in nonequilibrium statistical physics.

1. We consider a Hamiltonian system of N particles with quadratic interaction, defined by a positive definite matrix V. This is the worst system in the sense of convergence to equilibrium because of the existence of invariant tori. Nevertheless, it is proved [2] that if at least one of the particles is acted upon by an external random stationary force $f(t)$ with dissipation, then for almost all V the following convergence theorem holds: For any initial conditions, convergence to a unique invariant measure μ holds. In this case μ will be a Gibbs measure for a given system of N particles only if the stationary process $f(t)$ has no memory, so it is a white noise. The temperature depends on the variance of the white noise and the dissipation parameter. The natural hypothesis is that for systems with more complex interaction it will be the same, since the widespread view is that nonlinear Hamiltonian systems have (in general) better mixing than linear systems.

2. We consider the same Hamiltonian system, but the random effect on one isolated particle has a completely different character. Namely, at random moments of time, the sign of the velocity of the selected particle is replaced, that is, the energy-preserving transformation. In [1] it is proved that, for any initial states, convergence to the Liouville measure takes place on the corresponding energy surface.

3. The problem of breaking a time-varying chain of molecules, in which the extreme left particle is fixed, and on the extreme right a constant tensile external force acts. In [3], a procedure, called in physics the double scaling limit, was used to find the exact picture (in the parameter space) of the phase transition.

4. Transport flows of a large number of particles. In [4] and [5], the flow of particles on a straight line with a leader is considered, where the leader moves at will, and the motion of any other particle depends only on its distance to the previous particle. This system will no longer be Hamiltonian. The problem of optimal control of the stability is considered eliminating collisions of particles and increasing the flow density. A phase diagram with regions of stability, instability, and partial stability is obtained.

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A. A. Muromskaya (Moscow, Russia) — **The work model of a joint-stock insurance company using a dividend strategy with a step function of the barrier.**

Consider Sparre Andersen's model, according to which the capital of an insurance company paying dividends at time t has the form $X(t) = x + ct - \sum_{i=1}^{N(t)} X_i - D(t)$, $t \geqslant 0$ (see [1]). Here, c is the intensity of premium income, $N(t)$ is the renewal process, and $D(t)$ is the total dividends paid to the time moment t. Random variables $\{X_i\}$, denoting the size of claims, are independent and identically distributed. In addition, $\{X_i\}$ and the process $N(t)$ are also assumed to be independent. Dividends are paid in accordance to a barrier strategy with a barrier level $b(t)$ such that $b(t) = b_i$ on half-intervals of the form $t \in [T_{i-1}, T_i)$, $i \geqslant 1$, where T_i is the time of arrival of the ith claim, $T_0 = 0$. Within the framework of this model, an upper bound was obtained for the probability of ruin of a company (an analogue of Lundberg's inequality [2] for the case of dividend payments). An example of a strategy with a step function of the barrier is given, for which the estimate is less than 1. Under the additional condition that $N(t)$ is a Poisson process, a stronger inequality for the ruin probability is also proved.

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I. V. Pavlov (Rostov-on-Don, Russia) — **Stochastic analysis on deformed structures: Survey of results and main directions for further research.**[9]

This talk focuses on processes with discrete time, defined on structures more general than a standard stochastic basis. We call these structures deformed stochastic bases of the first and second kind. (The choice of terminology is argued in detail in [1].) Namely, let $(\Omega, \mathbf{F})$ be a filtered space with discrete time, where Ω is an arbitrary set and $\mathbf{F} = (\mathcal{F}_n)_{n=0}^{\infty}$ is an increasing sequence of σ-algebras. The family $\mathbf{Q} = (Q^{(n)}, \mathcal{F}_n)_{n=0}^{\infty}$ of probability measures $Q^{(n)}$, defined on $\mathcal{F}_n$, is called D1 — first kind deformation (respectively, D2 — second kind deformation) if $Q^{(n+1)} \,|\, \mathcal{F}_n \ll Q^{(n)}$ (respectively, $Q^{(n+1)} \,|\, \mathcal{F}_n \gg Q^{(n)}$) for any $n \in \mathbf{N} = \{0, 1, \dots\}$.

The fundamental role of classical martingales in various branches of mathematics (especially in financial mathematics) is well known. Using deformations, we introduce the concepts of deformed martingales of the first and the second kind. For any $n \in \mathbf{N}$ let random variables Z_n belong to $L_1(\Omega, \mathcal{F}_n, Q^n)$, and let $\mathbf{Q}$ be D1 (respectively, D2). The process $\mathbf{Z} = (Z_n, \mathcal{F}_n, Q^n)_{n=0}^{\infty}$ is called the DM1 — deformed martingale of the first (respectively, DM2 — second) kind if, for any $n \in \mathbf{N}$ $Q^{(n+1)}$-a.s. (respectively, $Q^{(n)}$-a.s.), $Z_n = E^{Q^{(n+1)}}[Z_{n+1} \,|\, \mathcal{F}_N]$. Analogously we define deformed supermartingales and submartingales (DSupM1, DSupM2, DSubM1, DSubM2), deformed local martingales (DLM1, DLM2), and deformed potentials (DP1, DP2).

A number of results published by the author in the past three years (co-authored with O. B. Nazarko) are analyzed and supplemented in the report: (1) Doob-type decomposition

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for DSubM1; (2) the Krikeberg-type decomposition for DM2 and the Riesz-type decomposition for DSupM2; (3) Doob-type optional sampling theorem for DSubM1 and DSubM2; (4) characterization of DLM1 in terms of deformed martingale transformations and deformed generalized martingales; (5) reduction of deformation of the first kind to weak deformations; (6) applications to financial mathematics; (7) concepts of deformed stochastic bases and deformed martingales in continuous time. The statements and proofs of all these results can be found in [2], [3], [4], [5], [6], and [7].

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E. L. Presman, V. I. Arkin, and A. D. Slastnikov (Moscow, Russia) — **The structure of the continuation set in the problem of optimal stopping of general one-dimensional diffusion.**[10]

Regular diffusion X_t $(t \geqslant 0)$, $X_0 = x$ with values in the interval I is considered. If the measures of velocity and killing have densities and the scale is twice differentiable, Itô diffusion is obtained. In [1] it was suggested consider necessary and sufficient conditions that the continuation set $C{=}\{x\colon\ g(x){<}V(x)\}$, where $V(x){=}\sup_\tau \mathbf{E}_x g(X_{\ tau})$, has some sort of structure. For a continuation set of the form $C{=}\{x \in I\colon\ x{<}p\}$ such conditions under additional assumptions on the payoff function and the presence of discounting were obtained in [1] for Itô diffusion and in [2] (conditions of another type) for general diffusion without killing, and formulated in [3] without additional assumptions on diffusion and the payoff function. We obtain necessary and sufficient conditions of the insular nature of continuation set: $C{=}\{x \in I\colon\ q{<}x{<}p\}$. The proof is based on the characterization of excessive functions and a method of modifying the payment function (see [4]).

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V. V. Rodochenko and O. E. Kudryavtsev (Rostov-on-Don, Russia) — **Application of the Wiener–Hopf factorization to the calculation of the risks of the intersection of price barriers in the Heston model.**[11]

To illustrate a method of risk management of a securities portfolio based on analysis and hedging of the risks of yielding prices for a fixed barrier, we consider in the Heston model a barrier option put with a barrier from below. Using Carr's randomization procedure and a suitable replacement [1] to eliminate the correlation between the price and variation processes, we get the opportunity to reduce the calculation of the contract-free price of the contract to a recurrent solution of a family of problems with stochastic variation.

Analogously [1], we use an approximation of the CIR variation process with the help of the Markov chain, after which we construct an approximation of the desired functional in the form of a family of problems with fixed variation, each of which can be solved with the help of the Wiener–Hopf factorization [2]. It is possible to generalize the method to the case of models admitting jumps.

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V. V. Shamraeva (Rostov-on-Don, Russia) — **Inequalities that ensure the fulfillment of interpolation properties of martingale measures.**[12]

Among martingale measures of the incomplete market, which generate an open interval of fair prices for the payment obligation, in the case of a countable probability space for a large number of (B, S)-markets, there are interpolation martingale measures that generate "more fair" prices. In this paper we consider martingale measures for one-step (B, S)-markets satisfying the weakened condition of noncoincidence of barycenters (WCNB; see [1]), which allows one to interpolate (with respect to arbitrary interpolating special Haar filtering) an incomplete market to a complete market. Consider the filtration $(\Omega, \mathbf{F})$, where $\mathbf{F} = (\mathcal{F}_0, \mathcal{F}_1)$, $\mathcal{F}_0 = \{\Omega, \varnothing\}$, and $\mathcal{F}_1$ is generated by splitting Ω into a countable number of atoms B_i, $i \in \mathbf{N} = \{1, 2, \dots\}$. We define the process $Z = (Z_n, \mathcal{F}_n)_{n=0}^{1}$, considered the discounted share price, and denote it by $Z_0 = a$, $Z_1|_{B_i} = b_i$. Suppose that $B_1 < b_2 < b_3 < b_4$; each of these numbers can be present in the sequence $B_5, b_6, \dots$ a finite or infinite number of times, and there are no other numbers in this sequence. Let such a market be unbiased, with $a \neq b_2$ and $a \neq b_3$. The report demonstrates a new technique for proving the existence of interpolation martingale measures, which makes it possible to obtain results more general than in [1]. It is based on the replacement of complex inequalities from the WCNB containing various vague subsets of $\mathbf{N}$ by simpler inequalities containing concrete components of martingale measures.

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S. Ya. Shatskikh and L. E. Melkumova (Samara, Russia) — **Geometry of conditional quantiles of multidimensional probability distributions.**[13]

For n-dimensional probability distribution $F_{1\dots n}(x_1,\dots,x_n)$ we consider the determinant (see [1])

$$\omega = \begin{vmatrix} dx_1 & \dots & dx_{n-1} & dx_n \\ 1 & \dots & \dot q_{n-1\,|\,1}^{(x_{n-1},x_1)}(x_1) & \dot q_{n\,|\,1}^{(x_n,x_1)}(x_1) \\ \dots & \dots & \dots & \dots \\ \dot q_{1\,|\,n-1}^{(x_1,x_{n-1})}(x_{n-1}) & \dots & 1 & \dot q_{n\,|\,n-1}^{(x_n,x_{n-1})}(x_{n-1}) \end{vmatrix}, \tag{1}$$

constructed from the derivatives of one-dimensional conditional quantiles $\dot q_{i|j}^{(x_i,x_j)}(x_j)$:

$$F_{i\,|\,j}\big(q_{i\,|\,j}^{(x_i^0,x_j^0)}(x_j)\,|\,x_j\big) = F_{i\,|\,j}(x_i^0\,|\,x_j^0),$$

where $F_{i\,|\,j}(x_i\,|\,x_j)$ is a conditional distribution function.

Theorem. *If for probability distribution $F_{1\dots n}(x_1,\dots,x_n)$ all k-dimensional conditional quantiles $q_{i|1\dots k}^{(\mathbf{x}^0)}(x_1,\dots,x_k)$, $i=k+1,\dots,n$, where*

$$F_{i\,|\,1\dots k}\Big(q_{i\,|\,1\dots k}^{(\mathbf{x}^0)}(x_1,\dots,x_k)\,|\,x_1,\dots,x_k\Big) = F_{i\,|\,1\dots k}(x_i^0\,|\,x_1^0,\dots,x_k^0),$$

have the property of reproducibility under the restriction to one-dimensional conditional quantiles (see [2]) and a minor, constructed on the intersection of the first k columns and k rows, starting from the second, of the determinant (1), differs from 0, then the surface in $\mathbf{R}^n$, given by conditional quantiles

$$\Big\{\big(x_1,\dots,x_k,q_{k+1\,|\,1\dots k}^{(\mathbf{x}^0)}(x_1,\dots,x_k),\dots,q_{n\,|\,1\dots k}^{(\mathbf{x}^0)}(x_1,\dots,x_k)\big)\Big\}, \tag{2}$$

is a k-dimensional solution of the Pfaff quantile equation

$$\omega = 0. \tag{3}$$

Hence, solving (3), we find k-dimensional conditional quantiles with respect to the two-dimensional marginal distributions of the original multidimensional distribution. When the Darboux class (see [2]) of the form ω equals $2(nk)-1$, the surface (2) is an integral manifold of (3) of maximum possible dimension passing through the point $(x_1^0,\dots,x_n^0)$.

As a statistical application of the theorem, an approach to the estimation of multivariate conditional quantiles of a distribution on two-dimensional observations is considered in the case when this distribution has the reproducibility property. It is shown that the volume of two-dimensional observations necessary for constructing an estimate of a multidimensional quantile within the framework of the proposed approach is substantially smaller than the volume of observations of the full dimension, necessary for constructing traditional estimates of multidimensional conditional quantiles. An example of constructing an estimate of a conditional quantile for an elliptically contoured distribution is considered.

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A. N. Shiryaev (Moscow, Russia) — **Randomness in probability.**

The subject of this overview is focused on the concept of *randomness* and, more specifically, how it would be possible to determine formally what is an *individual random sequence.*

According to A. N. Kolmogorov (IV Soviet–Japanese Symposium, 1982), "in everyday speech we call *random* those phenomena where we do not find *regularity*, which would allow us to accurately predict their results. Generally speaking, there is no reason to believe that a random phenomenon should have any certain probability. Therefore, we should have to distinguish between *proper accident* (as the lack of regularity) and *stochastic randomness* (which is an object of probability theory)."

In this report we adhere to the following plan:

2. Von Mises' frequency theory of probability.
3. Frequency stability, or stochasticity (Mises, Wald, Church, Kolmogorov, Loveland).
4. Typicality, or belonging to a set of effective measure unit (Martin-Löf, Levin, Shnorr).
5. Complexity, or randomness (Kolmogorov, Levin, Shnorr).
6. Unpredictability (Will, Uspensky).

I. M. Sonin and S. A. Molchanov (Charlotte, USA) — **Conditional expectations and transfusion from empty to empty.**

About 10 years ago Cherny and Grigoriev [1] obtained the following surprising result.

THEOREM. *Let $(\Omega, F, \mathbf{P})$ be a probability space without atoms and X, Y be bounded functions with the same distribution. Then for any $\varepsilon > 0$ there exists a sequence of σ-subalgebras $F_1, \dots, F_n \subseteq F$ such that for the sequence of functions $X_0 = X$, $X_1 = \mathbf{E}\,(X_0 \mid F_1)$, $X_2 = \mathbf{E}\,(X_1 \mid F_2), \dots, X_n = \mathbf{E}\,(X_{n-1} \mid F_n)$, the inequality $||X_n - Y|| \le \varepsilon$ holds.*

If a σ-algebra is defined by a finite or countable partition, then the conditional mathematical expectation of the function is nothing more than the averaging of this function with respect to the more "rough" decomposition. The partition of the interval $[0, 1]$ into $2n$ equal intervals is equivalent to the following "hydrostatic" problem. Suppose that there are $2n$ cups; the n left ones are filled with water, and n on the right are empty. The left and right cups can be joined together so that the liquid levels in them are aligned. The theorem mentioned is equivalent to the following assertion: if n is sufficiently large, then almost all the water can be pumped from left to right. Here we answer the two questions raised by this theorem. What is the optimal method of transfusion, and how much will remain for a fixed n? We describe all optimal transfusions and prove that the first term of the asymptotic in this problem has the form $2/\sqrt{n\pi}$. Another interpretation of the Cherny and Grigoriev theorem, without answers to these two questions, is given in [2].

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I. V. Tsvetkova (Rostov-on-Don, Russia) — **An algorithm for construction of interpolation martingale measures in the case of a countable probability space and finite-valued stock prices.**[14]

We consider a one-step $(\mathbf{B}, \mathbf{S})$-market, which is defined on $(\Omega, \mathbf{F})$, where $\mathbf{F} = (\mathcal{F}_k)_{k=0}^{1}$, $\mathcal{F}_0 = \{\Omega, \varnothing\}$, and $\mathcal{F}_1$ is a σ-algebra, generated by the partition Ω to the countable number of atoms B_i, $i \in N = \{1, 2, \dots\}$. Let $Z = (Z_k, \mathcal{F}_k)_{K=0}^{1}$ be an $\mathbf{F}$-adapted random process with values $Z_0(\Omega) = a$, $Z_1(B_I) = b_i$, $a \in \mathbf{Q}$, $b_i \in \mathbf{Q}$, $i \in \mathbf{N}$ ($\mathbf{Q}$ is a set of rational numbers). Suppose that among the elements of the sequence $\{b_i\}_{i=1}^{\infty}$ there are only r different

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($3 \leqslant r < \infty$) and at least two values with infinite multiplicity. The considered market is incomplete. Suppose that it is arbitrage free ($\inf_{1\leqslant i<\infty} b_i < a < \sup_{1\leqslant i<\infty} b_i$). The transition from incomplete markets to fulls provided with the help of special Haar interpolation, which presupposes the existence of martingale measures that satisfy a special interpolation property (see [1]). Based on the results presented in [2], an algorithm for computing such martingale measures is obtained.

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S. I. Uglich, I. V. Pavlov, and N. P. Krasiy (Rostov-on-Don, Russia) — **Numerical analysis of analytical results obtained in the study of quasilinear models with random priorities.**[15]

In this report, analytical results obtained in [1], as well as in the report by Krasny at the OTNA-2016 Conference in Rostov-on-Don, were used. In these papers an extremal problem with the following objective function is considered:

$$F(x) = E^P(F_1^{\alpha_1} F_2^{\alpha_2}), \quad \text{where } F_k(x) = \left(\sum_{i=1}^n a_i^k x_i + b^k\right) I_{\{\sum_{i=1}^n a_i^k x_i + b^k > 0\}}.$$

a_i^k, b^k are real numbers; $x = (x_1, x_2, \ldots, x_n) \in \mathbf{R}^n$; I is a characteristic function; and $\alpha_k = \alpha_k(\omega)$ are nonnegative random variables, called priorities, defined on $(\Omega, \mathcal{F}, P)$. In cases when random variables α_1 and α_2 are connected by the relation $\alpha_1 + \alpha_2 = 1$ or are independent and do not exceed one, the main result is formulated identically. The necessary condition is the following: the global maximum exists if there exists a number $c > 0$ such that $a_i^2 = -c \cdot a_i^1$. The parameter c determines the degree of opposition of the requirements of two systems with the objective functions F_1 and F_2. It is shown that for any $c > 0$ the maximum points of the function F are a hyperplane of the form $\sum_{i=1}^n a_i x_i = t^*$, where t^* is a root of some equation. The report shows the dependence of the maximum value F^* of the function F on the parameter c for different α_1 and α_2. Especially important for applications are cases where $F^*(c)$ has a global minimum (for example, when α_1 and α_2 are uniformly distributed).

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V. V. Ulyanov (Moscow, Russia) — **General approach leading to estimates of accuracy of approximation.**[16]

In the first part of our talk we give a brief review of recent results on estimates of accuracy of approximations for distribution of linear forms of random elements. We focus on the results for quadratic forms and almost quadratic forms, research that is motivated by asymptotic problems of mathematical statistics (see, for example, [1]). We give asymptotic results where we specify only the approximation order in n, where n is a number of random elements (a sample size), and in p, a dimension of random elements (observations) for p,

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compared to n. We also give nonasymptotic results when, instead of decreasing order, more informative inequalities for accuracy of approximation were proved. General nonlinear forms (see, for example, [2], [3], and [4]) also occurred, as a rule, in multidimensional statistics.

In order to obtain the results mentioned above, we use different methods. However, in [5] was proposed an approach permitting us a general sufficiently case to prove nonasymptotic results for nonlinear forms, including cases when as approximation we use asymptotic expansions, and estimates of accuracy of distributions are given in terms of Lyapunov ratios. In [5] is considered a class of real functions $h_n(\varepsilon, \dots, \varepsilon_n)$, $n \geqslant 1$, on $\mathbf{R}^n$, symmetric with respect to all possible permutations of its arguments and such that

$$h_{n+1}(\varepsilon_1, \dots, \varepsilon_j, 0, \varepsilon_{j+1}, \dots, \varepsilon_n) = h_n(\varepsilon_1, \dots, \varepsilon_j, \varepsilon_{j+1}, \dots, \varepsilon_n)$$

and

$$(\partial/\partial\varepsilon_j) h_n(\varepsilon_1, \dots, \varepsilon_j, \dots, \varepsilon_n)|_{\varepsilon_j=0} = 0 \quad \text{for all } j = 1, \dots, n.$$

If we consider a sequence of independent random elements X_j with generalized the distribution P, then we can choose

$$h_n = \mathbf{E}\, F(\varepsilon_1(\delta_{X_1} - P) + \cdots + \varepsilon_n(\delta_{X_n} - P));$$

i.e., h_n is a mean of a smooth functional F of a *weighted* empirical process with Dirac measures in $X_1, \dots, X_n$. In other words, h_n can be considered as a collection of "contributions" of random elements X_j. In the general case F depends on these measures nonlinearly. In this talk we show (see proofs in [5]) that if the "natural" moment conditions on the distribution X_1 are the indicated, then for pointed class of functions h_n there exists a "limit" function, and we can write asymptotic expansions of Chebyshev–Edgeworth type. Here the error of the estimate is given in $|\varepsilon|^d := \sum_{i=1}^n |\varepsilon_i|^d$. In this talk we discuss possible applications of the general approach, in particular in the central limit theorem for weighted sums, when with large probability with respect to a measure on an $(n-1)$-dimensional unit ball distributions of weighted sums are approximated by the normal distribution with accuracy of order $O(n^{-1})$. We consider also applications in asymptotic problems for distributions of U-statistics of order 2 and higher and in the central limit theorem for "free" probabilities.

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E. B. Yarovaya (Moscow, Russia) — **Stochastic evolution of a system of particles in a noncompact phase space: An approach using branching random walks.**[17]

Different phenomena arising in statistical physics, homopolymer theory, population dynamics, and other applications are often described in terms of the evolution of particle populations. Such models can be generalized in various directions, one of which is the assumption that the particles not only produce offspring or die, forming a branching environment, but also migrate under the influence of some random law. The central problem in such models is

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the study of the evolution of processes in time, depending on the structure of the medium and the spatial dynamics of the particles. Less studied in this case are processes when the space in which particles transport occurs is unlimited, and it can be continuous, as in homopolymer theory, for example, or discrete, as in models of the dynamics of cellular populations. In this sense, “universal” describes branching random walks with continuous time with respect to multidimensional lattices — stochastic processes that combine the properties of a branching process and a random walk. The main problems in the study of the limiting behavior of branching random walks are related to the existence of phase transitions as the various parameters of the particle system change, the properties of limiting distribution of particle populations, and velocity and shape of the propagation of their front. Naturally, the solution of these problems depends to a large extent on a number of factors that affect the properties of a branching random walk, among then the randomness of the branching medium, its heterogeneity, the number and relative positioning of the sources of multiplication and death of particles at lattice points, and also such properties of the random walk as symmetry or symmetry breaking, and the finiteness or infinity of the variance of the jumps. The report discusses the results obtained in some space-time evolutionary models of branching random walks; see, for example, [1], [2], [3], [4], [5], [6], [7], [8], and their applications.

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V. G. Zadorozhnii (Voronezh, Russia) — **The stabilization of linear systems by Gaussian random noise.**

Consider a linear system of differential equations $dx/dt = \varepsilon(t,\omega)Ax + f(t,\omega)$, where $x \in \mathbf{R}^n$, ε is a scalar random Gaussian process with mathematical expectation $\mathbf{E}\,\varepsilon$ and covariation function $b(s_1,s_2) = \mathbf{E}\,(\varepsilon(s_1)\varepsilon(s_2)) - \mathbf{E}\,(\varepsilon(s_1))\,\mathbf{E}\,(\varepsilon(s_2))$, ω is a random event, and $f(t,\omega)$ is a random vector process.

A solution of the system $x(t,x_0)$ with initial condition $x(t_0,x_0) = x_0$ is called (compare with [1]) average stable if for any $\varepsilon > 0$ there exists $\delta(\varepsilon) > 0$ such that for any ξ, satisfying condition $\|\mathbf{E}\,\xi - \mathbf{E}\,x_0\| < \delta(\varepsilon)$, the condition $\sup_{t\ge t_0} \|\mathbf{E}\,(x(t,\xi)) - \mathbf{E}\,(x(t,x_0))\| < \varepsilon$ is fulfilled. If in this case $\|\mathbf{E}\,(x(t,\xi)) - \mathbf{E}\,(x(t,x_0))\| \to 0$ as $t \to +\infty$, then the solution is called average asymptotically stable.

The considered linear system is stable, asymptotically stable, or average unstable [1] if and only if

$$\left\| \exp\left(A \int_{t_0}^{t} \mathbf{E}\,(\varepsilon(s))\, ds + \frac{A^2}{2} \int_{t_0}^{t} \int_{t_0}^{t} b(s_1,s_2)\, ds_1\, ds_2 \right) \right\|$$

is limited for $t \in [t_0,\infty)$, tends to zero as $t \to +\infty$, or is unbounded for $t \in [t_0,\infty)$, respectively.

Hence we obtained the conditions under which the considered system becomes asymptotically stable, although the system $dx/dt = Ax$ is unstable.

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M. V. Zhitlukhin (Moscow, Russia), **K. A. Borovkov, Yu. S. Mishura, and A. A. Novikov — On maxima of Gaussian processes and their approximations.**[18]

This talk is devoted to estimates of mathematical expectations of maxima of Gaussian processes and their approximations by discrete time processes. Namely, we consider Gaussian processes X_t with zero mean, which satisfy the inequalities

$$(*)\qquad C_1|t-s|^{H_1} \leqslant (\mathbf{E}\,(X_t - X_s)^2)^{1/2} \leqslant C_2|t-s|^{H_2} \qquad \text{for all } t,s \geqslant 0$$

with some constants $C_1, C_2 > 0$ and $H_1, H_2 \in (0,1)$. A fundamental example of a process of this type is a fractal Brownian motion B_t^H, a Gaussian process with parameter $H \in (0,1)$, with the covariance function

$$\mathbf{E}\,(B_t^H B_s^H) = \frac{1}{2}\,(t^{2H} + s^{2H} - |t-s|^{2H}).$$

In applications the important problem is seeking (or estimating) mathematical expectations of Gaussian processes, $\mathbf{E}\max_{t\leqslant 1} X_t$; in particular, it allows us to investigate the asymptotics of the distribution function of the maximum. However, in a general case for processes with the structure $(*)$ it is not possible to find the exact mathematical expectation. Thus, we state several estimates for the given value, and also, bearing in mind the numerical approximations, we obtain estimates for errors of discrete approximation with the help of the value $\mathbf{E}\max_{0\leqslant i\leqslant n} X_{i/n}$.

The main results are the proofs of the following inequalities.

1. For the process X_t, satisfying $(*)$,

$$\frac{C_1 L_1}{\sqrt{H_1}} \leqslant \mathbf{E}\max_{0\leqslant t\leqslant 1} X_t \leqslant \frac{C_2 L_2}{\sqrt{H_2}}$$

holds, where $L_1 = 1/(4\pi e \log 2)$ and $L_2 = (15/4)\sqrt{(2\pi/\log^3 2)}$.

2. If X_t satisfies the right-hand side of inequality $(*)$, then for any $n \geqslant 2^{1/H_2}$,

$$\mathbf{E}\max_{0\leqslant t\leqslant 1} X_t - \mathbf{E}\max_{0\leqslant i\leqslant n} X_{i/n} \leqslant \frac{2C_2\sqrt{\log n}}{n^{H_2}}\left(1 + \frac{4}{n^{H_2}} + \frac{0.0074}{(\log n)^{3/2}}\right) \leqslant \frac{7C_2\sqrt{\log n}}{n^{H_2}}.$$

3. For fractal Brownian motion B_t^H and any $0 < H_1 < H_2 < 1$, the estimate

$$\mathbf{E}\max_{0\leqslant i\leqslant n} B_{i/n}^{H_1} - \mathbf{E}\max_{0\leqslant i\leqslant n} B_{i/n}^{H_2} \leqslant \sqrt{\frac{H_2 - H_1}{eH_1}\,\log n}$$

holds.

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