Afanasyev V.I. (Moscow, Russia). **Boundary problems for a random walk in a random environment.**

Let $\{X_k, k \geq 0\}$ be a random walk in a random environment and the random environment is a sequence of independent and identically distributed random vectors (p_i, q_i), $i \in \mathbf{Z}$, where $p_0 + q_0 = 1$, $p_0 > 0$, $q_0 > 0$. By definition it means that for a fixed random environment the sequence $\{X_k, k \geq 0\}$ is a discrete Markov chain starting from 0 with the set of states $\mathbf{Z}$ and transition probabilities p_{ij} such that $p_{i,i+1} = p_i$, $p_{i,i-1} = q_i$, $i \in \mathbf{Z}$.

Suppose that

$$\mathbf{E} \ln \frac{q_0}{p_0} = 0, \qquad \mathbf{E} \ln^2 \frac{q_0}{p_0} := \sigma^2, \qquad \sigma^2 \in (0, +\infty). \tag{1}$$

Let $T_n = \min\{k \geq 1 : X_k = n\}$, where $n \in \mathbf{Z}$. The random variable $\ln T_n$, as $n \to \infty$, is well studied. In particular, limit theorems are established for it as in the case when the condition (1) is valid so and in the case when the value $\mathbf{E} \ln^2 (q_0/p_0)$ is infinite (see, for example, [1]).

Consider a two-boundary problem about the first exit of the sequence $\{X_k, k \geq 0\}$ from the interval $(-\lfloor an \rfloor, \lfloor bn \rfloor)$, where $a, b > 0$. The following results are valid (see [2]).

Theorem 1. *If the condition* (1) *is valid and* $a, b > 0$, *then*

$$\lim_{n \to \infty} \mathbf{P}\left(T_{\lfloor bn \rfloor} < T_{-\lfloor an \rfloor}\right) = \frac{2}{\pi} \arctan \sqrt{a/b}.$$

Consider for $x, y > 0$ and $k, l \in \mathbf{Z}$ the triangle

$$S_{k,l}(x, y) = \left\{(u, v) \in \mathbf{R}^2 : u/x + v/y \leq k + l + 1,\ u \geq kx,\ v \geq ly\right\}.$$

Let $G(x, y) = \bigcup_{k,l \in \mathbf{Z}} S_{4k,2l+1}(x, y)$, $D(x, y) = \bigcup_{k,l \in \mathbf{Z}} S_{4k+2,2l+1}(x, y)$. Suppose that ξ_1, ξ_2 are independent random variables with standard normal distribution. Let

$$F(x, y) = \mathbf{P}((\xi_1, \xi_2) \in G(x, y)) - \mathbf{P}((\xi_1, \xi_2) \in D(x, y)).$$

Theorem 2. *If the condition* (1) *is valid and* $a, b > 0$, *then for any* $x > 0$

$$\lim_{n \to \infty} \mathbf{P}\left(\frac{\ln T_{\lfloor bn \rfloor}}{\sigma \sqrt{n}} < x \,\middle|\, T_{\lfloor bn \rfloor} < T_{-[an]}\right) = 2\pi \frac{F\left(x/\sqrt{b}, x/\sqrt{a}\right)}{\arctan\left(\sqrt{a/b}\right)}.$$

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