Albosaily S., Pergamenshchikov S. (Rouen, France). **Optimal investment and consumption for Ornstein–Uhlenbeck spread financial markets with power utility**

We study an investment/consumption optimization problem for financial markets of Brownian motion type generated by the differences in risky financial assets for investor who can trade in a one risk-free bond and multiple stocks. The goal of the investor is to allocate money that his expected utility from terminal wealth is maximized. The model of the financial market "Spread"we use is driven by Ornstein-Uhlenbeck processes and it is given by:

$$\mathrm{d}X_t^v = (rX_t - \sum_{i=1}^d \alpha_i < A_1 S_t >_i - c_t)\mathrm{d}t + \sum_{i=1}^d \alpha_i \sigma_i \mathrm{d}W_i(t)\,,$$

This model was proposed by Boguslavsky and Boguslavskaya [2] for a pure optimal investment of one dimensional problem. By solving the Cauchy problem for non-linear equations from O. Ladyženskaja [3] and by using the Leray-Schauder fixed point theorem. The existence and uniqueness for the following Hamilton-Jacobi-Bellman equation is shown.

$$z_t(\varsigma,t) + \frac{1}{2}\sum_{i,j=1}^d \frac{(\tilde{a}_{ij} s_i z_x - z_{xs_i}\sigma_i^2)^2}{\sigma_i^2 |z_{xx}|} + \frac{1}{2}\sum_{i=1}^d \sigma_i^2 z_{s_i s_i}$$

$$+ \sum_{i,j=1}^2 a_{ij} s_j z_{s_i} + rxz_x + (1-\gamma)\left(\frac{z_x}{\gamma}\right)^{\frac{\gamma}{\gamma-1}} = 0,$$

where $z(\varsigma, T) = \varpi x^\gamma$. The optimal investment and consumption strategies are obtained and respectively given by

$$\check{\alpha}_i^0(\varsigma,t) = \frac{\tilde{a}_{ij} s z_x(\varsigma,t)}{\sigma^2 z_{xx}(\varsigma,t)} - \frac{z_{xs_j}(\varsigma,t)}{z_{xx}(\varsigma,t)} = \check{\beta}(s,t)x, \quad \text{and} \quad \check{c}^0(\varsigma,t) = \left(\frac{z_x(\varsigma,t)}{\gamma}\right)^{\frac{1}{\gamma-1}} = \check{G}(s,t)x,$$

where

$$\check{\beta}(s_j,t) = \frac{1}{1-\gamma}\Big(s_j g_j(t) + Y_{s_j}(s,t) - \frac{\hat{\kappa}_j}{\sigma^2} s_j\Big) \quad \text{and} \quad \check{G}(s,t) = \varpi^{\frac{1}{\gamma-1}} U(s,t,Y(s,t)).$$

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