Lisovskii D. I. (Moscow, Russia) — **Sequential hypothesis testing problem for stationary Gauss-Markov processes.**

We consider continuous in probability stationary Gauss-Markov processes. Due to the classical J. Doob's result [1], it is well known that such a class coincides with one of stationary Ornstein-Uhlenbeck processes. It is assumed that there are two hypotheses to distinguish

$$H_0\colon dX_t = \theta(\mu - X_t)dt + \sigma dB_t, \quad X_0 \sim \mathcal{N}(\mu, \sigma^2/(2\theta)),$$
$$H_1\colon dX_t = \gamma(\mu - X_t)dt + \sigma dB_t, \quad X_0 \sim \mathcal{N}(\mu, \sigma^2/(2\gamma)),$$

where $\sigma > 0$, $\mu \in \mathbb{R}$ is a mean function of X, the parameters $\theta, \gamma > 0$ stand for the speed of mean-reversion of the observable process, and $B = (B_t)_{t \geqslant 0}$ is a standard Brownian motion being independent of an initial value X_0 (all defined processes and random variables are assumed to be given on some probability space $(\Omega, \mathcal{F}, \mathsf{P})$.

Following A. Wald, we will discuss the sequential scheme $\Delta = \Delta(\tau, d)$ of hypothesis testing characterized by the following pair: a stopping moment τ which is a Markov time adopted to the natural filtration $\mathbb{F}^X$ generated by the observable process X and a function of the final decision d being $\mathcal{F}_\tau^X$-measurable random variable that can take only two different values each of which will be identified with a decision in favour of hypothesis H_0 or H_1. In accordance with R. Lipser and A. Shiryaev [2], a decision rule $\Delta^* = \Delta(\tau^*, d^*)$ is said to be optimal in a class of all sequential schemes provided the probabilities of wrong terminal decision are given and fixed if it minimizes Kullback-Leibler divergence in this class.

In our work [5], we investigate the SPRT (Sequential Probability Ratio Test) test which is known [3, 4] to be optimal for a number of models. However, it looses its optimality property in the problem formulated above and appears to be asymptotically optimal only. To be more specific, the SPRT test is shown to be asymptotically optimal when either the probabilities of a wrong terminal decision tend to zero or the mean-reversion speed parameters tend to infinity preserving the distance between them.

This talk is based on the joint work with Albert Shiryaev (Moscow).

REFERENCES

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